

Lecture 4

Flow Patterns and Wind Forces

Tokyo Polytechnic University
The 21st Century Center of Excellence Program

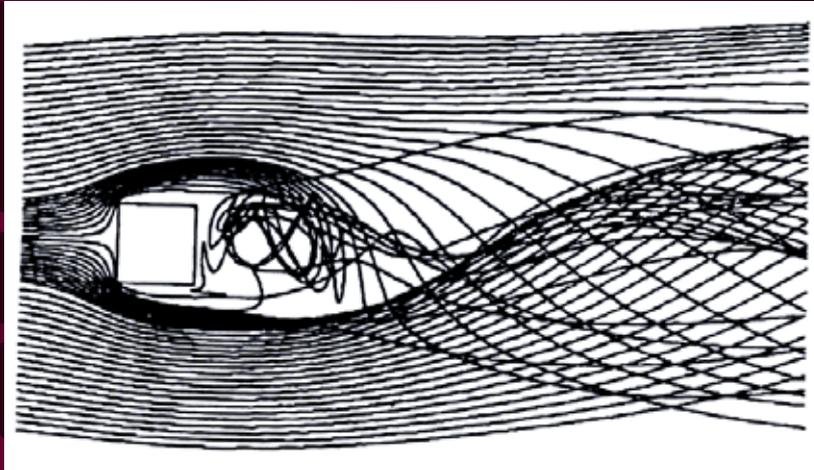
Yukio Tamura

Flows Around Bluff Bodies

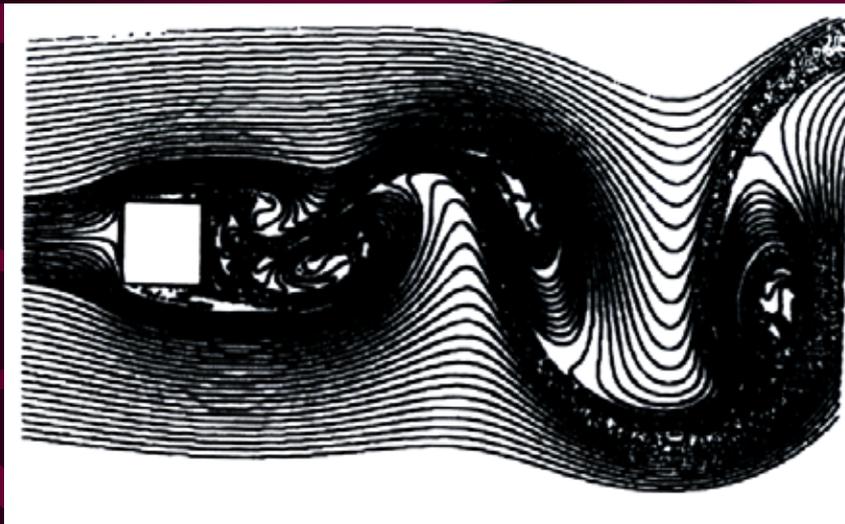
Flow Patterns Around Bluff Bodies



Stream Lines



Flow pattern around a square prism (Particle paths, 2D, Uniform flow, CFD)

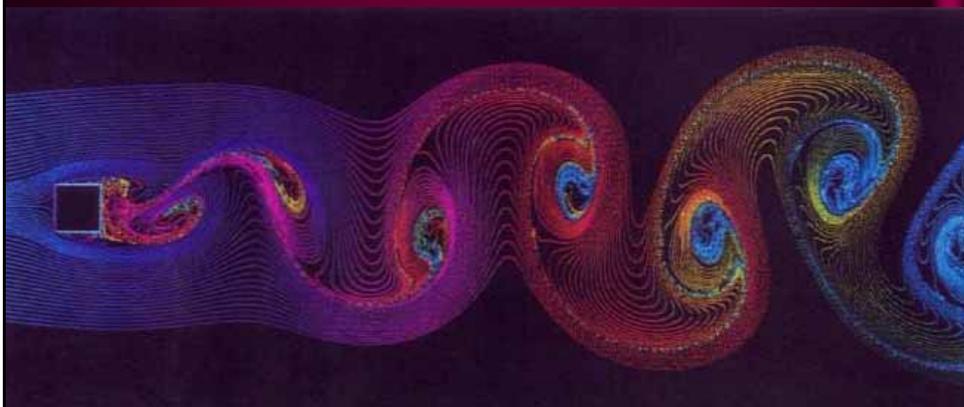


Flow pattern around a square prism (Streak Lines, 2D, Uniform Flow, CFD)

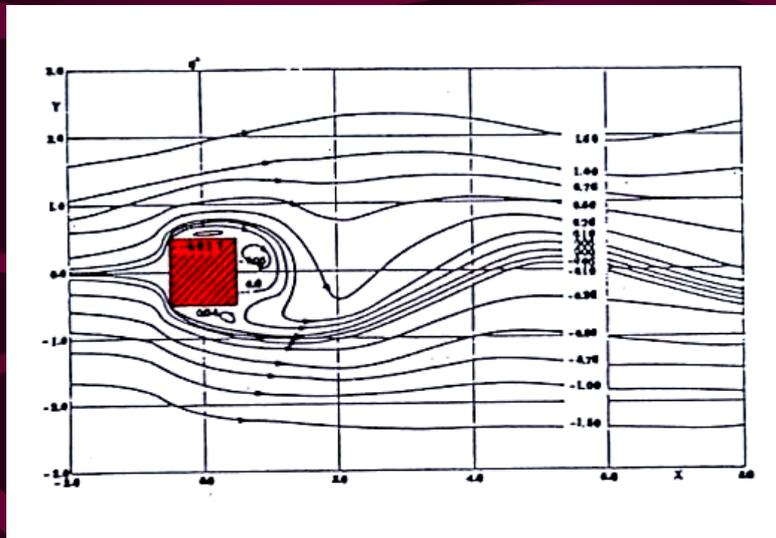


**Flow pattern between building models
(Streak Lines)**

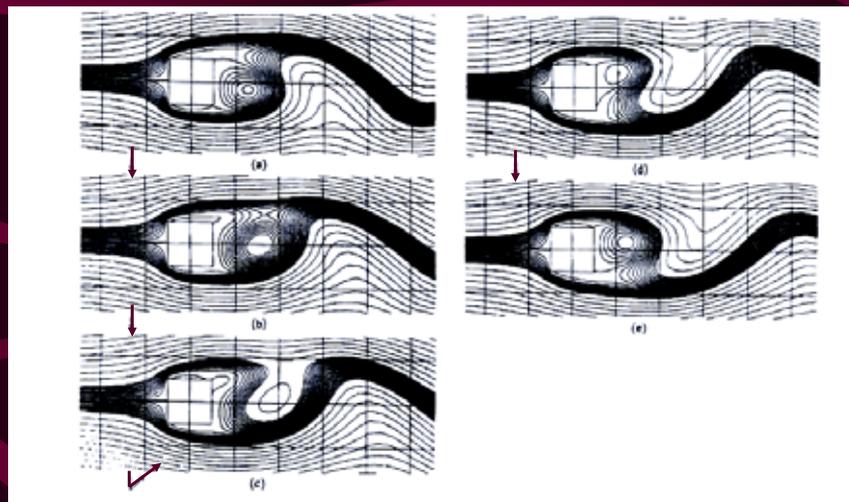
K. Shimada



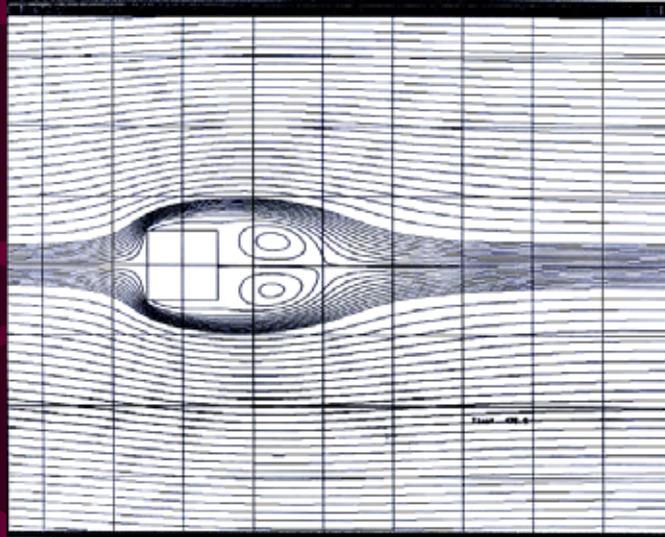
**Periodic vortices shed from a square prism
(Streak Lines, CFD)**



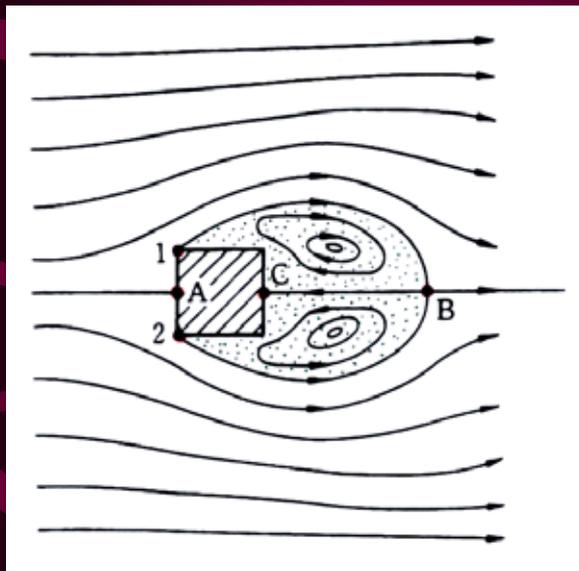
Flow pattern around a square prism
(Stream lines, Wind tunnel, by T. Mizota)



Temporal variation of flow pattern around a square prism
(Stream lines, 2D, Uniform flow, CFD)

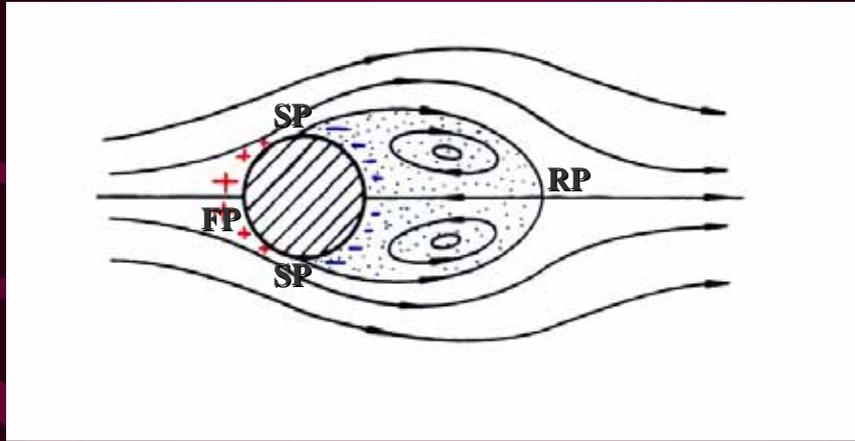


Temporally averaged flow pattern around a square prism (Stream lines, 2D, Uniform flow, CFD)

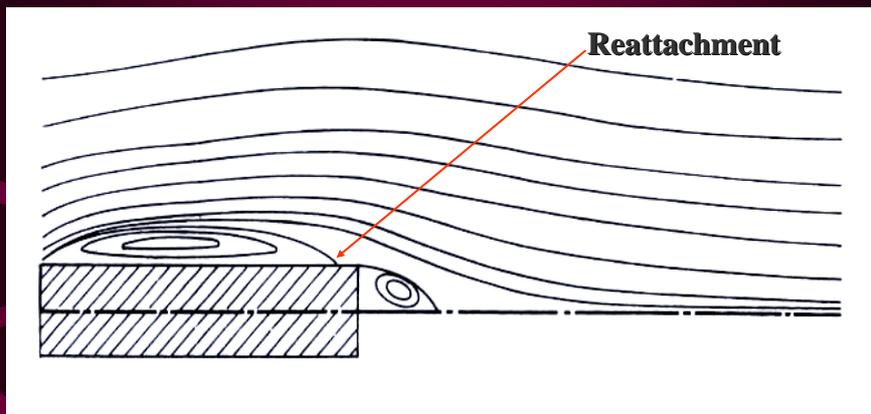


- A: Front stagnation point**
- B: Rear stagnation point**
- 1, 2: Separation point**

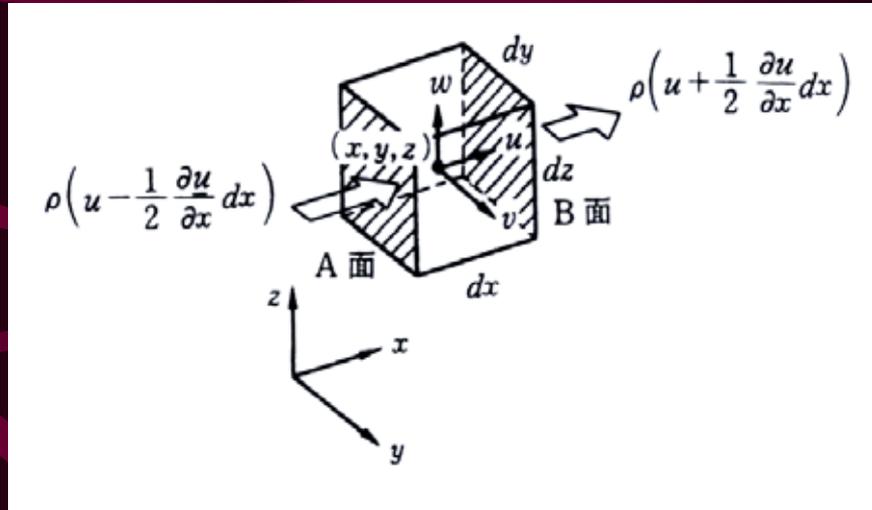
Temporally averaged flow pattern around a square prism



Temporally averaged flow pattern around a circular cylinder



Temporally averaged flow pattern around a rectangular cylinder with a large side ratio



Inflow and outflow into an infinitesimal hexahedron

Inflow through the $(dy \times dz)$ plane:

$$\begin{aligned} & \rho \left(u - \frac{1}{2} \frac{\partial u}{\partial x} dx \right) dydz - \rho \left(u + \frac{1}{2} \frac{\partial u}{\partial x} dx \right) dydz \\ &= -\rho \frac{\partial u}{\partial x} dx dy dz \end{aligned} \quad (1)$$

Inflow through the $(dz \times dx)$ plane:

$$= -\rho \frac{\partial v}{\partial y} dx dy dz \quad (2)$$

Inflow through the $(dx \times dy)$ plane:

$$= -\rho \frac{\partial w}{\partial z} dx dy dz \quad (3)$$

Law of Conservation of Mass

(Incompressible Fluid) : Eq.(1)+Eq.(2)+Eq.(3)=0

$$-\rho \frac{\partial u}{\partial x} dx dy dz - \rho \frac{\partial v}{\partial y} dx dy dz - \rho \frac{\partial w}{\partial z} dx dy dz = 0$$

Equation of Continuity:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

$$(\text{div } \vec{v} = 0)$$

■ Vorticity Vector $\vec{\omega}$:

$$\vec{\omega} = (\xi, \eta, \zeta) = \text{rot } \vec{v}$$

$$\xi = \frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}$$

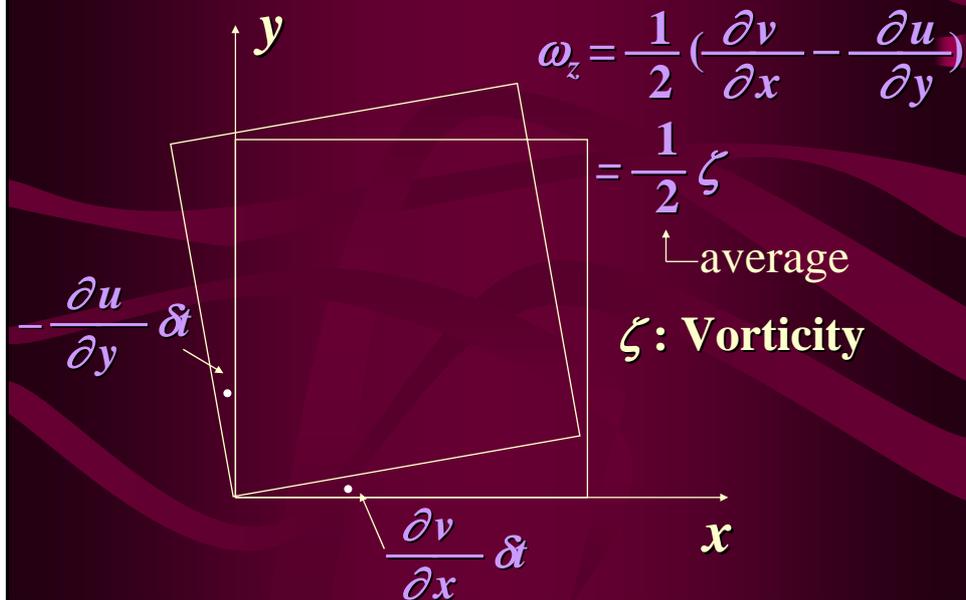
$$\eta = \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}$$

$$\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$$

$\xi, \eta, \zeta = 2\omega_x, 2\omega_y, 2\omega_z$: Vorticity

$\omega_x, \omega_y, \omega_z$: Rotational angular velocity
about x, y and z axes

Angular Velocity of Rotation



■ Non-viscous and Irrotational Fluid

$$\vec{\omega} = \text{rot } \vec{v} = 0$$

■ Velocity potential ϕ can be assumed:

$$u = \frac{\partial \phi}{\partial x}, \quad v = \frac{\partial \phi}{\partial y}, \quad w = \frac{\partial \phi}{\partial z}$$

■ **Equation of Continuity:**

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

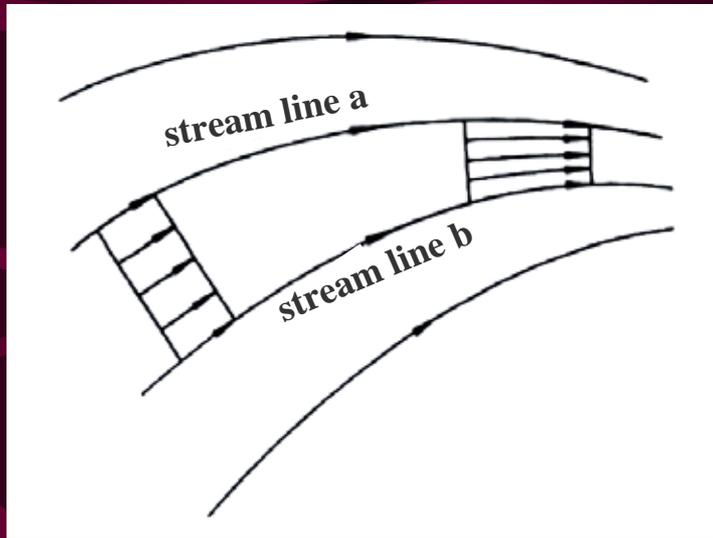
$$u = \frac{\partial \phi}{\partial x}, \quad v = \frac{\partial \phi}{\partial y}, \quad w = \frac{\partial \phi}{\partial z}$$

ϕ : Velocity potential

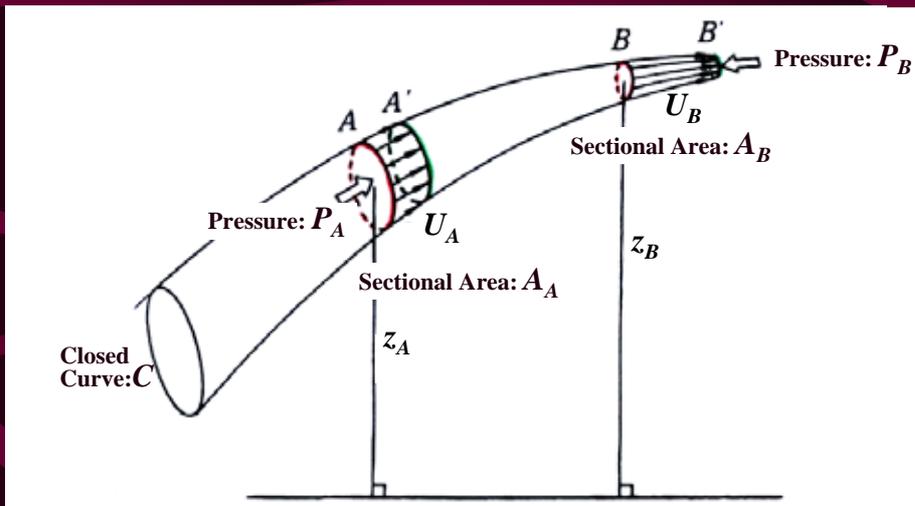
■ **Laplace Equation:**

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0$$

**Flow Pattern
and Pressure**



Stream line and flow velocity



Stream tube and flow velocity

Incompressible Flow (Law of conservation of mass)

$$\rho A_A U_A = \rho A_B U_B = m$$

m : Air mass of inflow and outflow portions

Increase of kinetic energy during unit time

$$\frac{m}{2} (U_B^2 - U_A^2) \quad (1)$$

Increase of potential energy during unit time

$$mg (z_B - z_A) \quad (2)$$

Work done by pressure difference during unit time

$$(P_A A_A)U_A - (P_B A_B)U_B \quad (3)$$

ρ : Air density, U_i : Wind speed at point i
 P_i : Pressure at point i , g : Gravity acceleration
 z_i : Altitude of point i

For the same stream tube:

$$\begin{aligned} & \frac{1}{2} \rho U_A^2 + P_A + \rho g z_A \\ &= \frac{1}{2} \rho U_B^2 + P_B + \rho g z_B \end{aligned}$$

ρ : Air density
 U_i : Wind speed at point i
 P_i : Pressure at point i
 g : Gravity acceleration
 z_i : Altitude of point i

Eq.(1)+Eq.(2)= Eq.(3)

Bernoulli's Equation (Steady / Ideal flow)

$$\frac{1}{2} \rho U^2 + P + \rho g z = H_T \text{ (constant)}$$

- ρ : Air density
- U : Wind speed
- P : Pressure
- g : Gravity acceleration
- z : Altitude
- H_T : Total pressure

(On Stream

Bernoulli's equation:

$$\frac{1}{2} \rho U^2 + P_S = H_T \text{ (constant)}$$

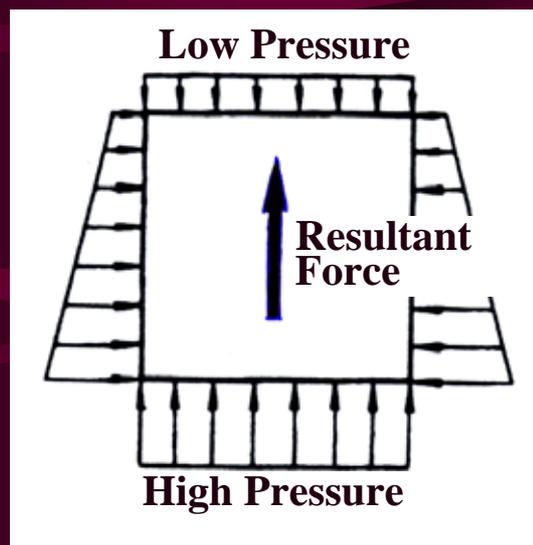
- ρ : Air density
- U : Wind speed
- P_S : Static pressure = $P + \rho g z$
- $\frac{1}{2} \rho U^2$: Dynamic pressure
- H_T : Total pressure

Bernoulli's equation:

$$\frac{1}{2} \rho U^2 + P_S = H_T \text{ (constant)}$$

U : Large (Small)

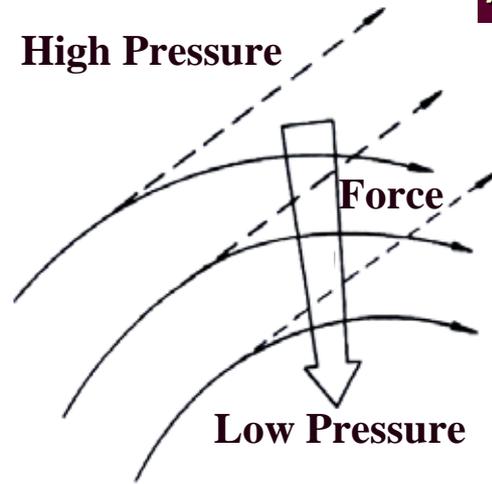
P_S : Small (Large)



Pressure distribution and resultant force acting on a fluid cell

$$\rho \frac{U^2}{r} + \frac{\partial P}{\partial n} = 0$$

High Pressure



Force

Low Pressure

Curved stream line and
pressure gradient

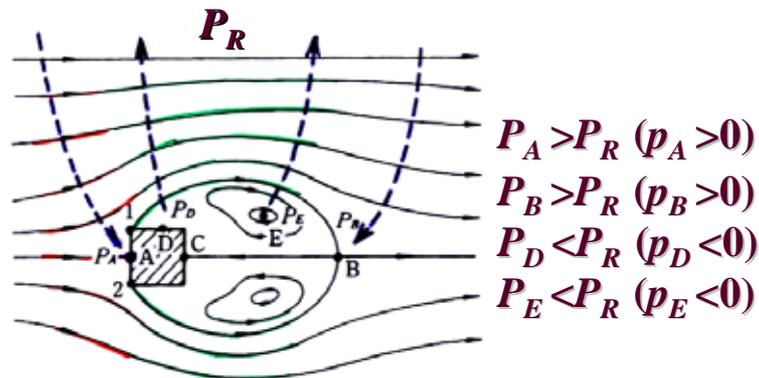
■ Wind Pressure at point i

$$p_i = P_i - P_R$$

P_i : Pressure at point i

P_R : Pressure at reference point R far away from the body, where there is no effect of the body on the flow field

Wind Pressure : $p_i = P_i - P_R$



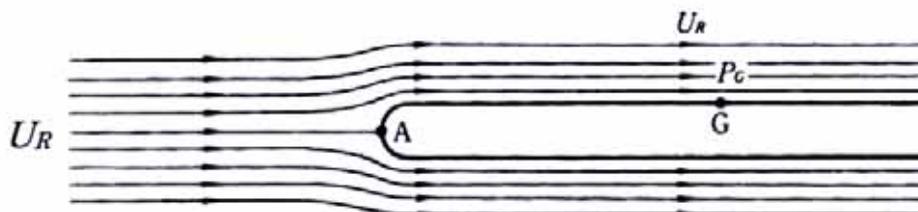
Stream lines around a square prism and spatial variation of pressure

$$p_A = P_A - P_R = P_A - P_G =$$

$$= (1/2)\rho U_R^2 \quad \text{: Dynamic pressure (Velocity pressure) of the reference point}$$

$$U_R = \sqrt{2p_A / \rho} \quad \text{(Reference wind speed)}$$

Anemometer



Flow around Pitot static tube and pressure

Navier-Stokes Equation

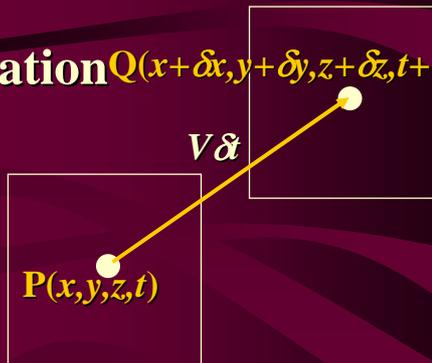
Substantial Differentiation $Q(x+\delta x, y+\delta y, z+\delta z, t+\delta t)$

Velocity components

$$u = u(x, y, z, t)$$

$$v = v(x, y, z, t)$$

$$w = w(x, y, z, t)$$



Displacement of a small element of fluid at point P during an infinitesimal interval δt

$$\delta x = u(x, y, z, t) \delta t$$

$$\delta y = v(x, y, z, t) \delta t$$

$$\delta z = w(x, y, z, t) \delta t$$

} Eq.(a)

Changing rate ($\delta f / \delta t$) of a property f

$$f + \delta f = f(x + \delta x, y + \delta y, z + \delta z, t + \delta t)$$

$$= f(x, y, z, t) + \frac{\partial f}{\partial x} \delta x + \frac{\partial f}{\partial y} \delta y + \frac{\partial f}{\partial z} \delta z + \frac{\partial f}{\partial t} \delta t + O(\delta t^2)$$

$\delta t \rightarrow 0$, substituting Eq.(a)

$$\frac{Df}{Dt} = \frac{\partial f}{\partial t} + u \frac{\partial f}{\partial x} + v \frac{\partial f}{\partial y} + w \frac{\partial f}{\partial z}$$

Substantial Acceleration

$$\frac{Du}{Dt} = \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z}$$

Changing rate ($\delta f / \delta t$) of a property f

$$f + \delta f = f(x + \delta x, y + \delta y, z + \delta z, t + \delta t)$$

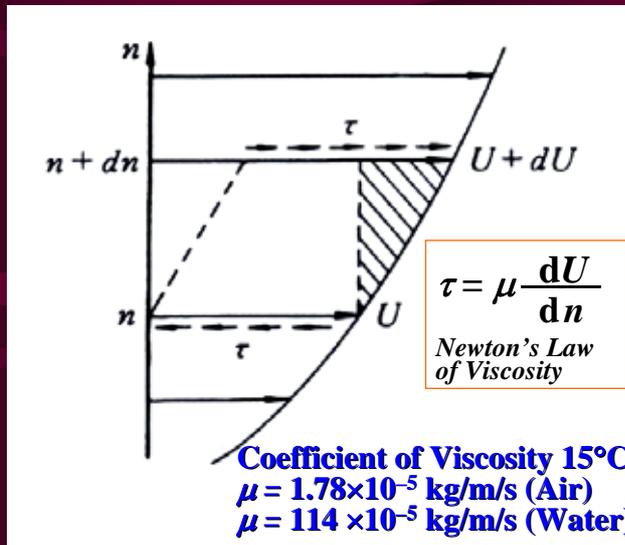
$$= f(x, y, z, t) + \frac{\partial f}{\partial x} \delta x + \frac{\partial f}{\partial y} \delta y + \frac{\partial f}{\partial z} \delta z + \frac{\partial f}{\partial t} \delta t + O(\delta t^2)$$

$\delta t \rightarrow 0$, substituting Eq.(a)

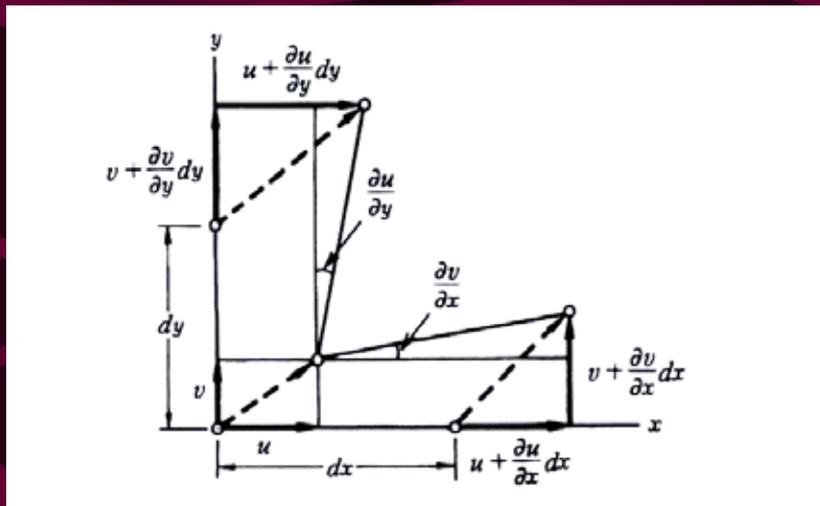
$$\frac{Df}{Dt} = \frac{\partial f}{\partial t} + u \frac{\partial f}{\partial x} + v \frac{\partial f}{\partial y} + w \frac{\partial f}{\partial z}$$

Substantial Acceleration

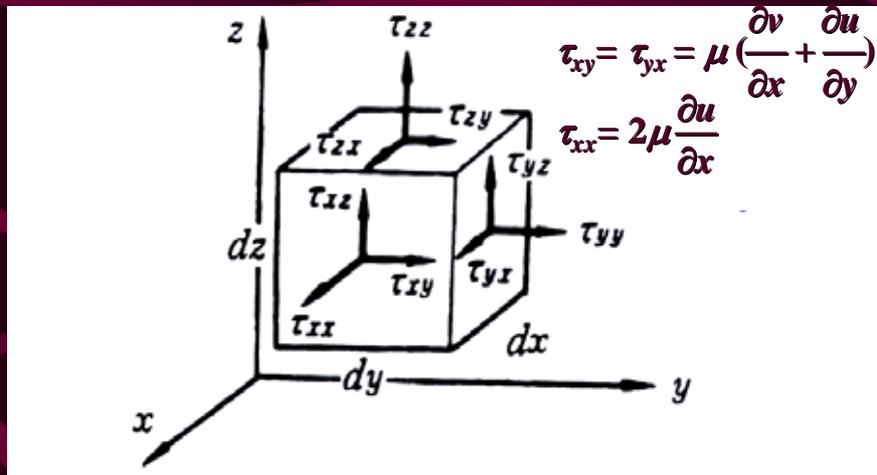
$$\frac{Du}{Dt} = \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z}$$



Velocity gradient and viscous stress (Newtonian fluid)



Shear deformation velocity in xy - plane



Shear stresses acting on an infinitesimal hexahedron of fluid

■ **Total shear forces acting x-direction:**

$$\left(\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} \right) dx dy dz$$

■ Navier Stokes Equations:

Convective acceleration

$$\rho \left[\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right]$$

Substantial acceleration
Local (Instantaneous) acceleration

$$= -\frac{\partial p}{\partial x} + \mu \left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right]$$

Pressure gradient Viscous stress

■ Navier Stokes Equations:

$$\rho \left[\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right]$$

$$= -\frac{\partial p}{\partial x} + \mu \left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right]$$

$$\rho \left[\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} \right]$$

$$= -\frac{\partial p}{\partial y} + \mu \left[\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right]$$

$$\rho \left[\frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} \right]$$

$$= -\frac{\partial p}{\partial z} + \mu \left[\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right]$$

■ **Non-dimensional expression of Navier Stokes Equations:**

$$\frac{\partial u^*}{\partial t^*} + u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*} + w^* \frac{\partial u^*}{\partial z^*} = -\frac{\partial p^*}{\partial x^*} + \frac{1}{Re} \left[\frac{\partial^2 u^*}{\partial x^{*2}} + \frac{\partial^2 u^*}{\partial y^{*2}} + \frac{\partial^2 u^*}{\partial z^{*2}} \right]$$

$$x^* = \frac{x}{L}, \quad y^* = \frac{y}{L}, \quad z^* = \frac{z}{L}, \quad t^* = \frac{tU}{L}, \quad p^* = \frac{p}{\rho U^2}$$

$$u^* = \frac{u}{U}, \quad v^* = \frac{v}{U}, \quad w^* = \frac{w}{U}, \quad Re = \frac{\rho UL}{\mu}$$

■ **Reynolds Number**

$$Re = \frac{\rho UL}{\mu} = \frac{UL}{\nu}$$

$$= \frac{\rho L^3 U^2 / L}{L^2 \mu U / L}$$

$$= \frac{\text{Inertial Force}}{\text{Viscous Force}}$$

$$\approx 7 \times 10^4 L U \quad (\text{Air})$$

(m/s) Reference Speed
(m) Reference Length

$$\approx 9 \times 10^5 L U \quad (\text{Water})$$

$T = 15^\circ\text{C}, P = 1013\text{hPa}$

$\rho = 1.22 \text{ kg/m}^3$

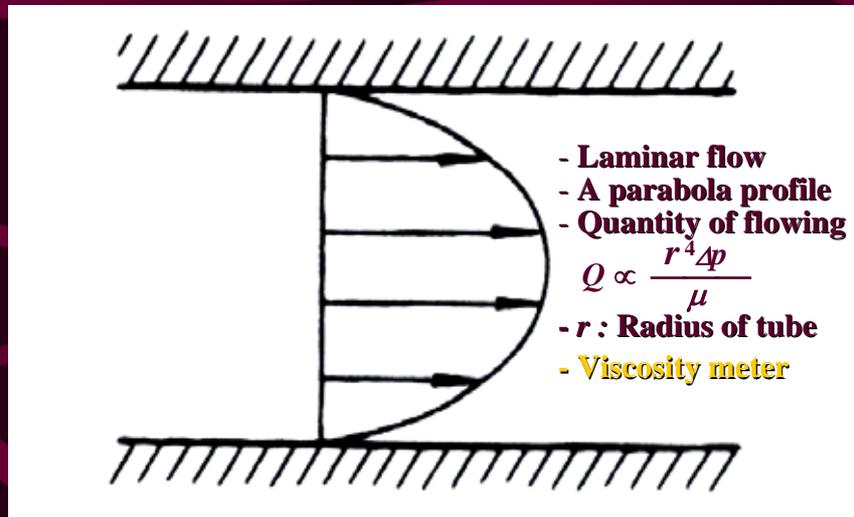
$\mu = 1.78 \times 10^{-5} \text{ kg/m/s (Air)}$

$\mu = 114 \times 10^{-5} \text{ kg/m/s (Water)}$

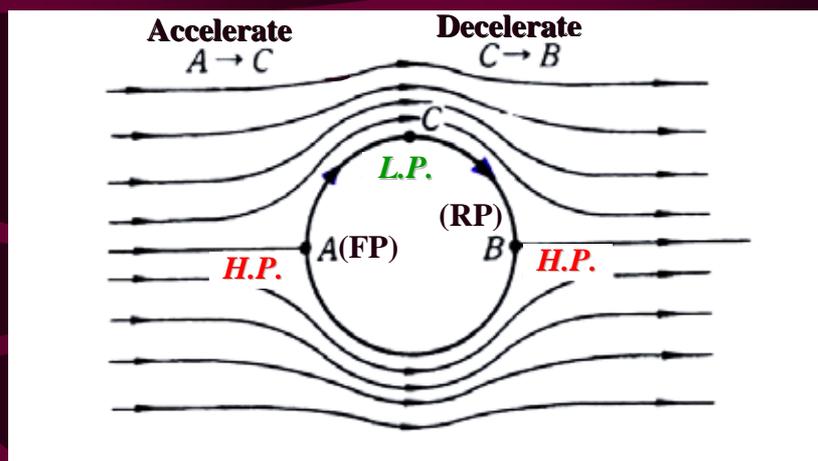
Dynamic Viscosity

$\nu = \mu/\rho = 1.45 \times 10^{-5} \text{ m}^2/\text{s (A)}$

$\nu = \mu/\rho = 1.14 \times 10^{-6} \text{ m}^2/\text{s (W)}$

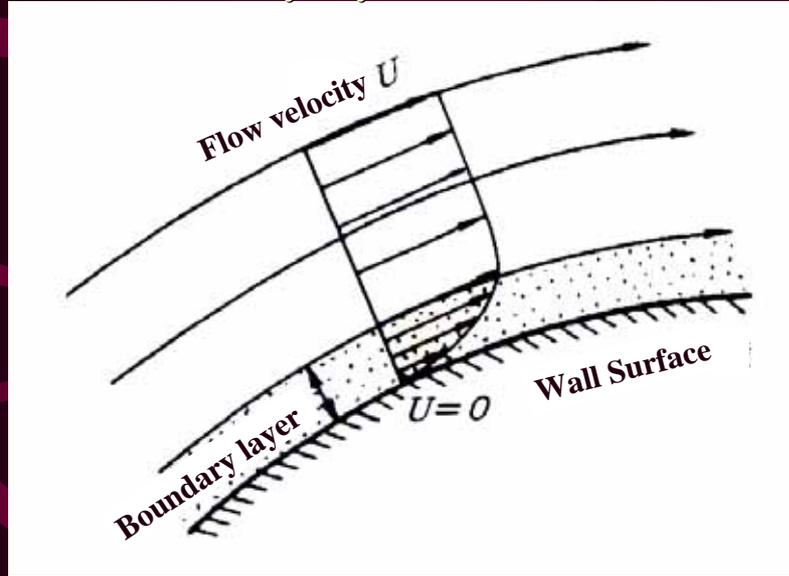


Hagen-Poiseuille flow in a straight circular tube

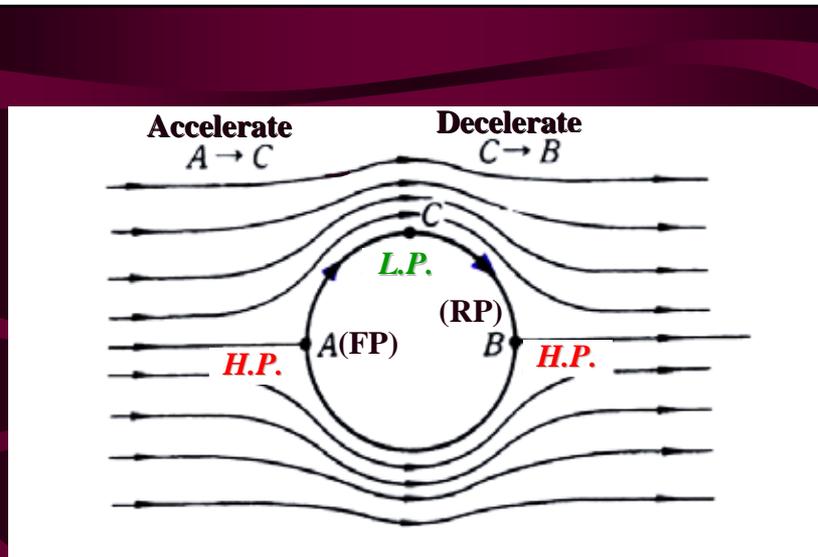


Flow around a circular cylinder in an ideal flow (Non-viscous)

Surface Boundary Layer

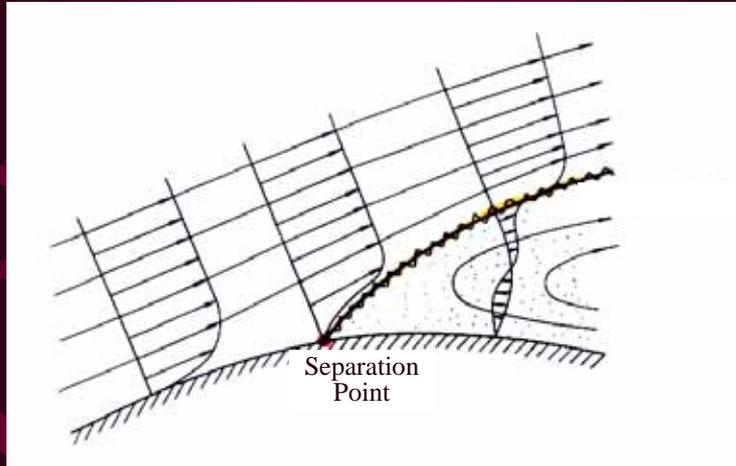


Flow near surface of body



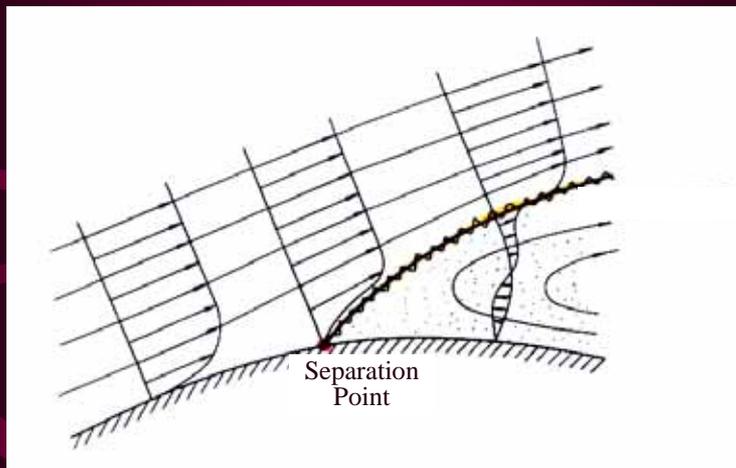
Flow around a circular cylinder in an ideal flow (Non-viscous)

- Separated shear layer
- Vortex sheet
- Shear layer instability

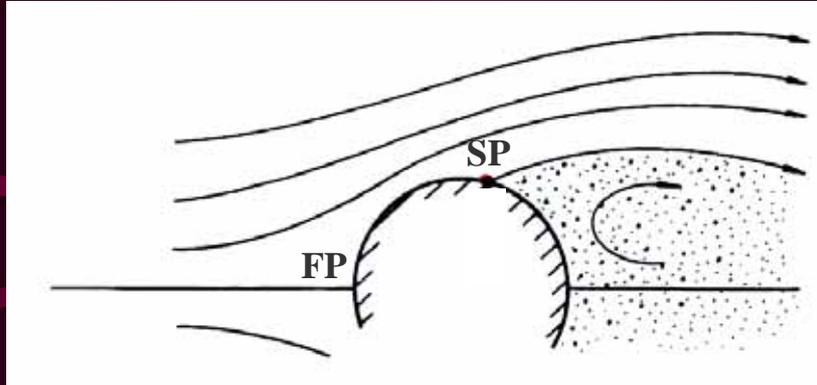


Flow near the separation point

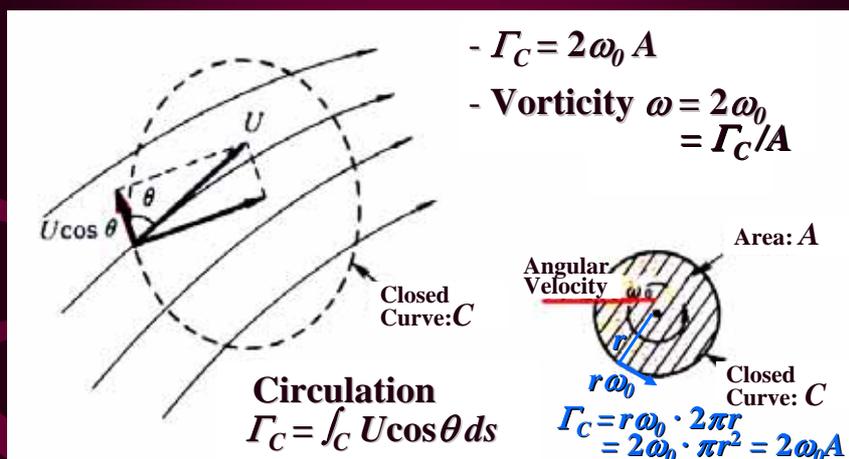
- Separated shear layer
- Vortex sheet
- Shear layer instability



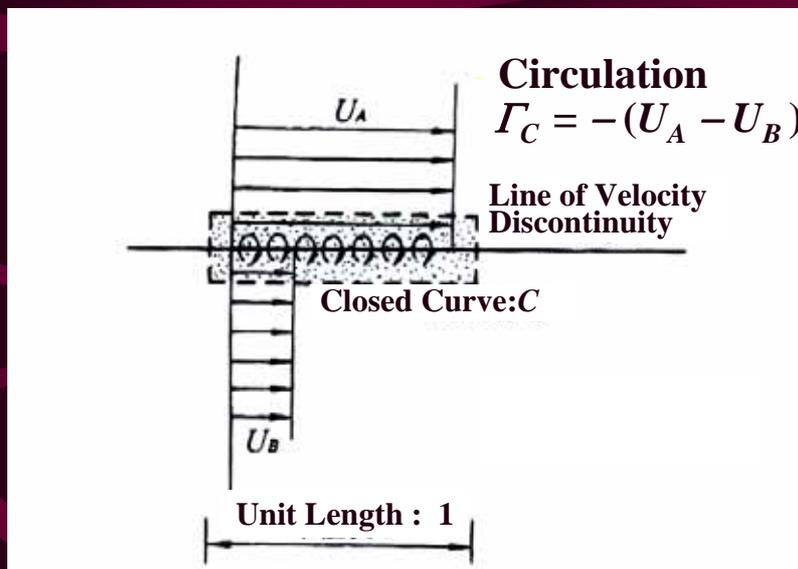
Flow near the separation point



**Flow separation from body surface
(Viscous Flow)**

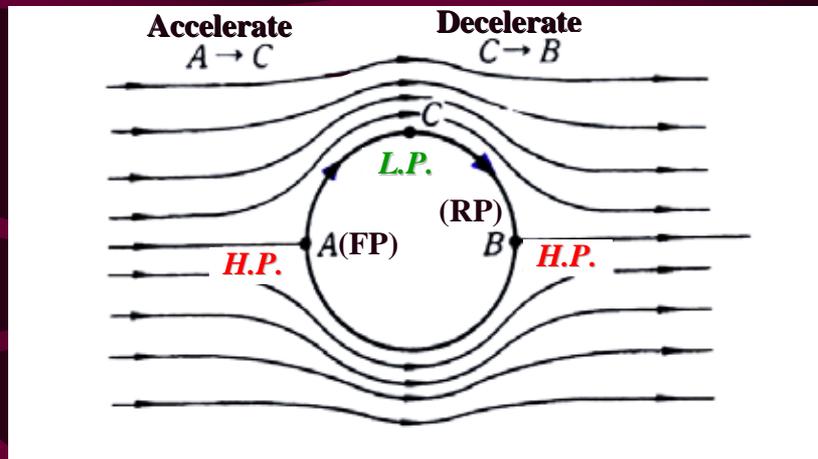


**(a) Definition of circulation (b) Rigid vortex and Circulation
Circulation and vorticity**



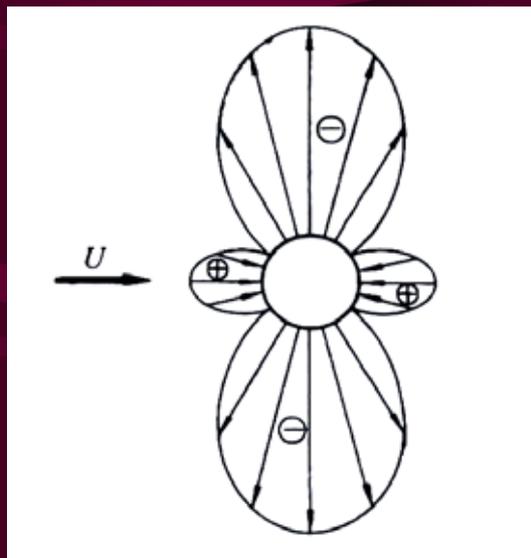
Circulation along a line of velocity discontinuity

Causes of Wind Forces

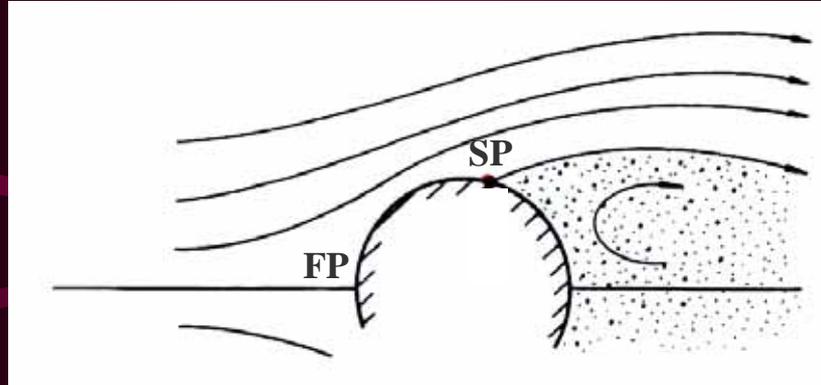


Flow around a circular cylinder in an ideal flow (Non-viscous)

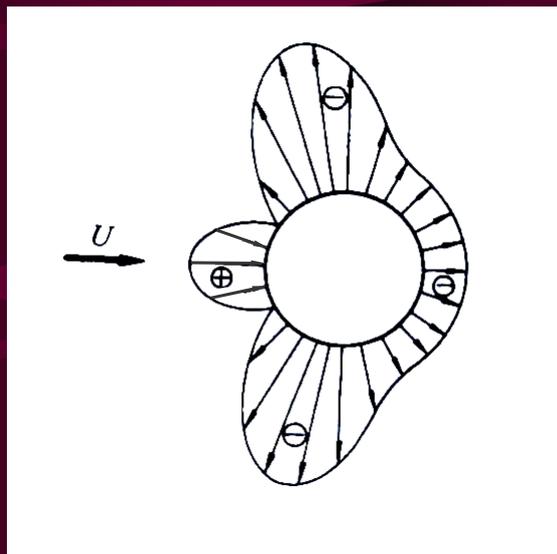
- D'Alembert's Paradox



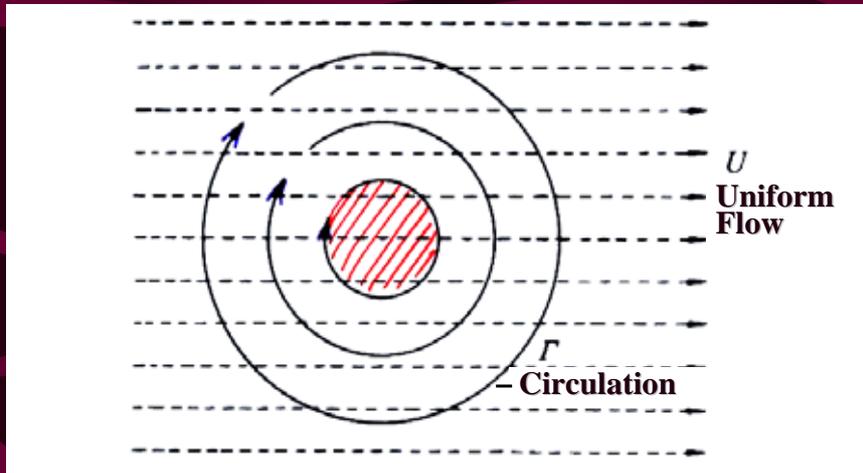
Pressure distribution on a circular cylinder in the ideal non-viscous flow



**Flow separation from body surface
(Viscous Flow)**

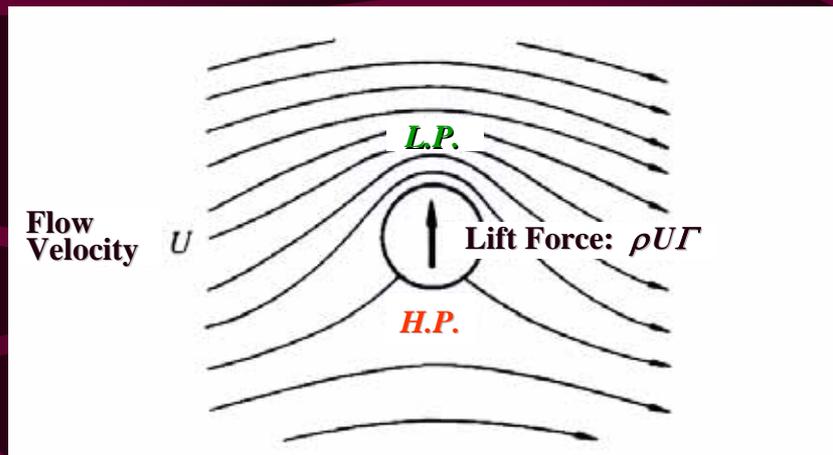


**Pressure distribution on a circular cylinder in
actual viscous flow**



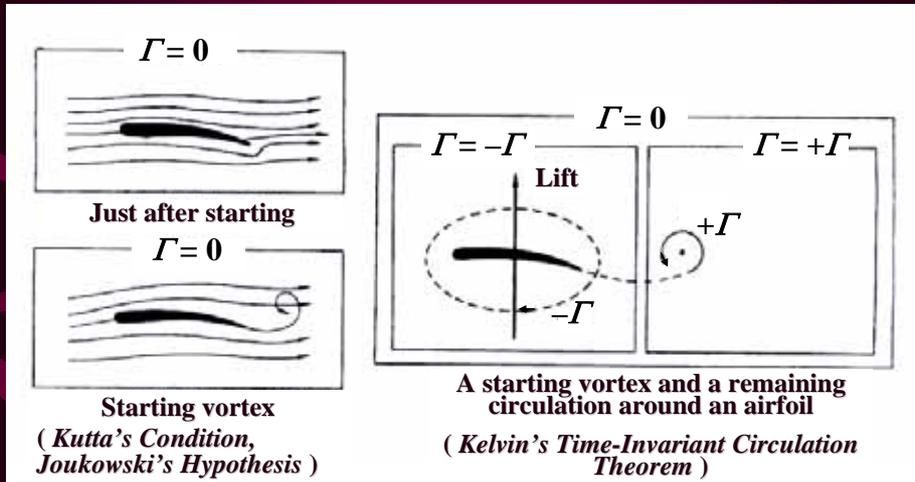
Superimposition of a rotational flow around a circular cylinder with a uniform flow

Kutta-Joukowski's Theorem

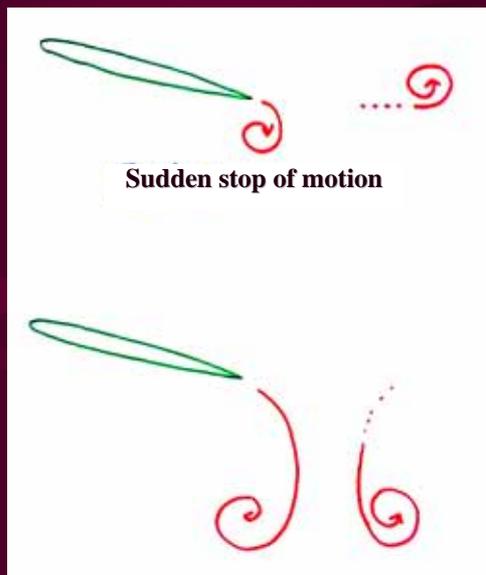


Stream lines around a circular cylinder in a uniform ideal flow superimposed on a clockwise circulation

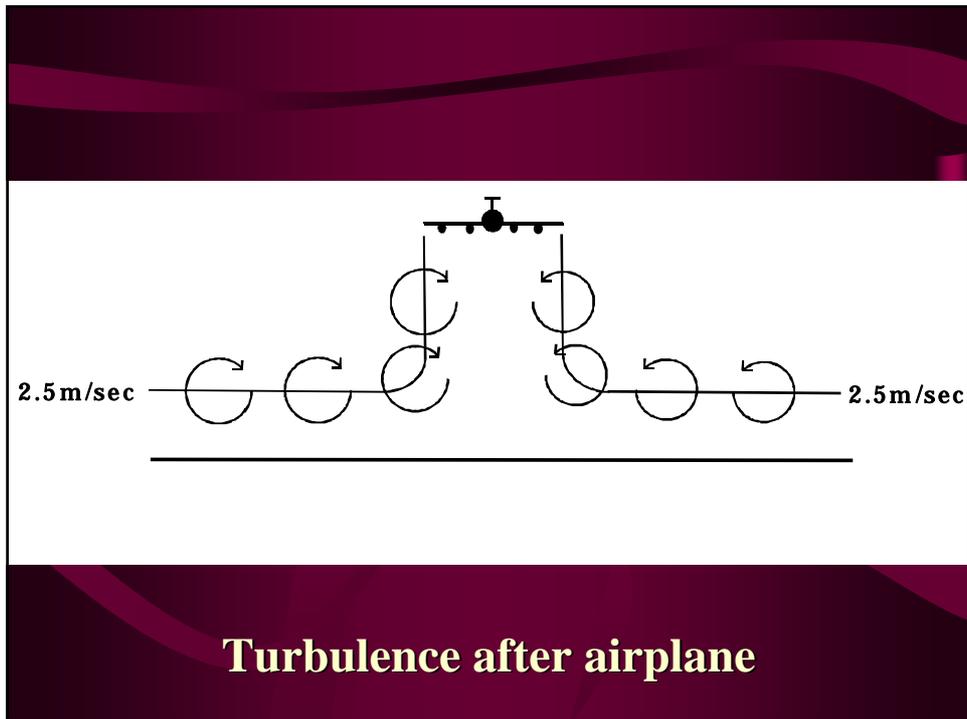
- Starting vortex



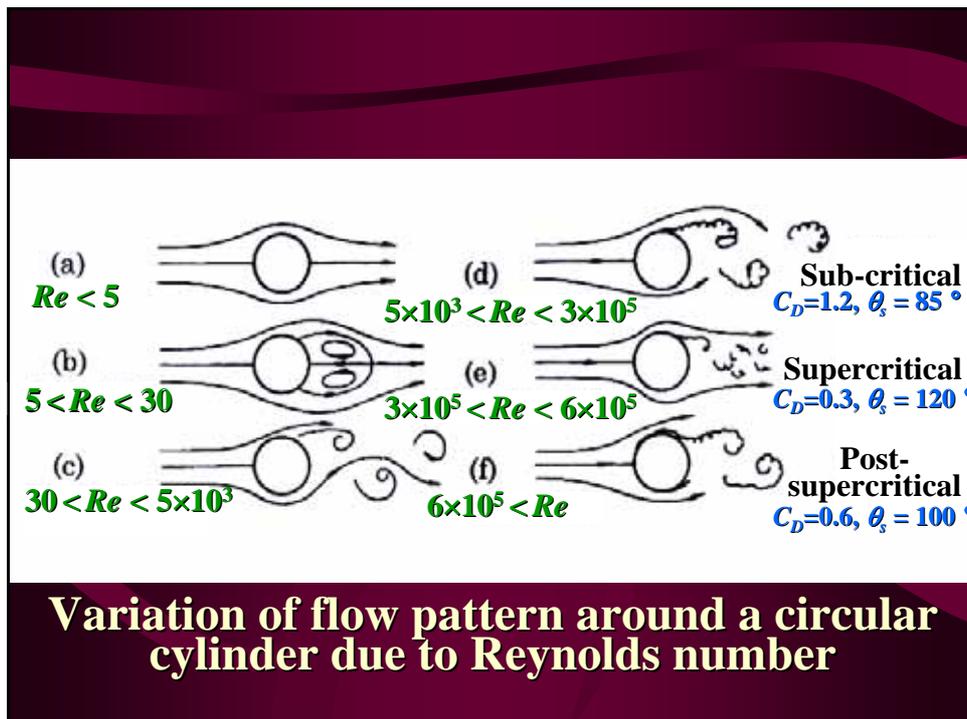
A vortex generated when an airfoil starts to move in a stationary flow



Release of a vortex stuck to an airfoil by sudden stop of motion



Reynolds Number and Flow Patterns

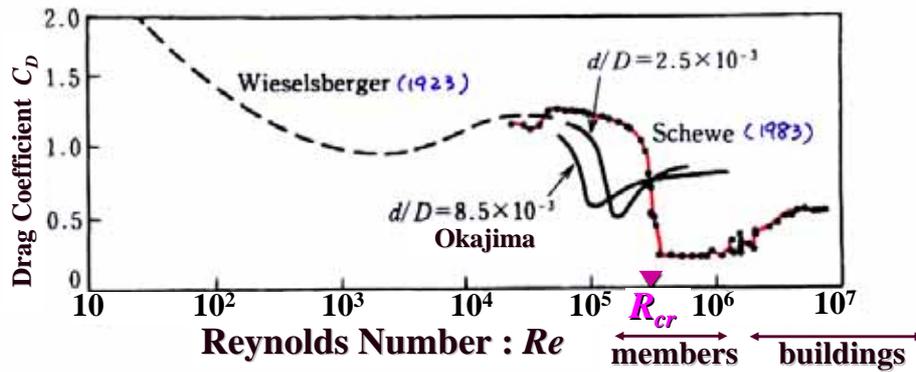


Important Regimes for Structures

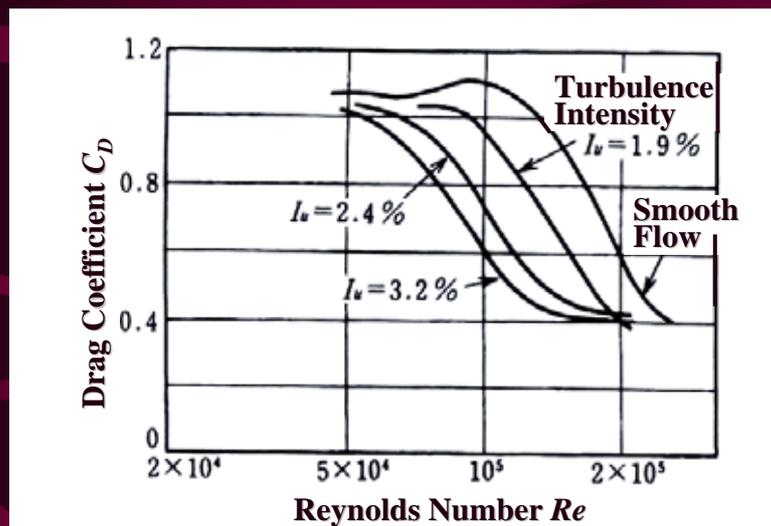
- **Sub-critical** $5 \times 10^3 < Re < 3 \times 10^5$
 Surface boundary layer : **Laminar**
 Separation points: $\theta_s = 85^\circ$
 Separated shear layer: **Turbulent in wake (Widest wake)**
 Drag Coefficient : $C_D = 1.2$
 Vortices : **Very periodic**
- **Supercritical** $3 \times 10^5 < Re < 6 \times 10^5$
 Surface boundary layer : **Laminar \rightarrow Turbulent $\theta > 85^\circ$**
 Separation points: $\theta_s = 120^\circ$
 Separated shear layer: **Narrow Wake-width**
 Drag Coefficient : $C_D = 0.3$
 Vortices : **Lose periodicity**
- **Post-supercritical** $6 \times 10^5 < Re$
 Surface boundary layer : **Fully turbulent**
 Separation points: $\theta_s = 100^\circ$
 Separated shear layer: **Wider wake-width**
 Drag Coefficient : $C_D = 0.6$ ($Re \approx 4 \times 10^6$)
 Vortices : **Periodic**

R_{cr} : Critical Reynolds Number

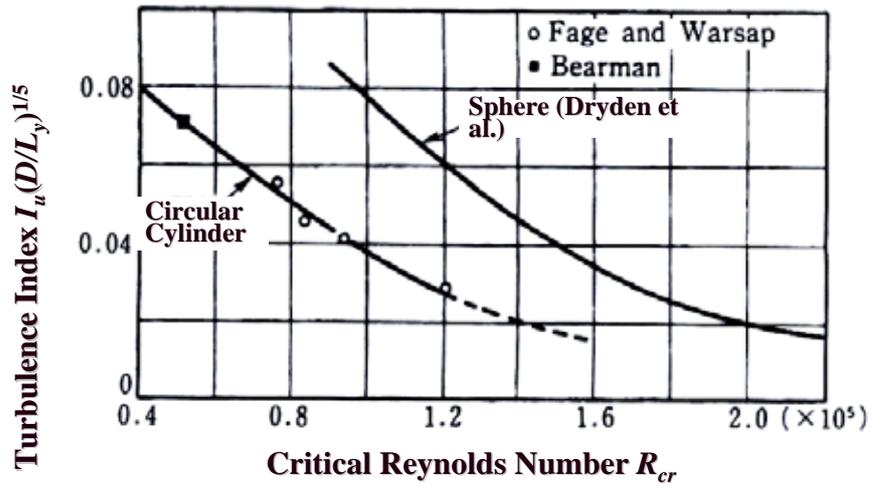
- d : Diameter of particles attached to the surface
- D : Diameter of a circular cylinder



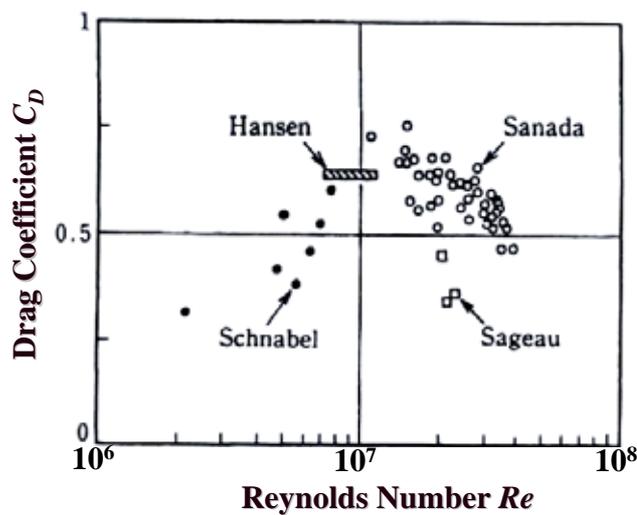
Variation of drag coefficient of a circular cylinder due to Reynolds number



Variation of mean drag coefficient of a circular cylinder with turbulence intensity and Reynolds number (2D)



Variation of the critical Reynolds numbers of a sphere and a circular cylinder with turbulence index $I_u(D/L_y)^{1/5}$ (Bearman, 1968)



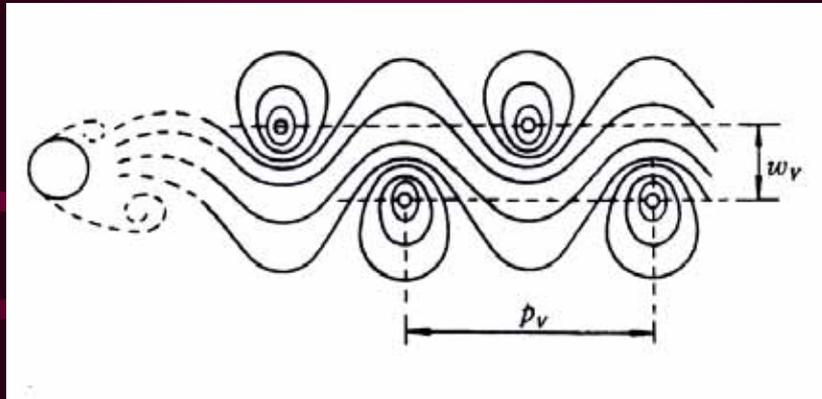
Field data of mean drag coefficients of actual circular structures (chimneys etc.)

Vortices Shed From Bluff Bodies

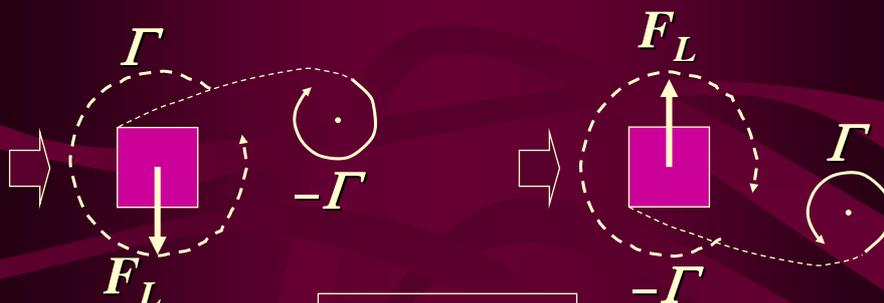


Karman vortices shed from a square prism

$$w_v / p_v = 0.281$$



Karman vortex streets



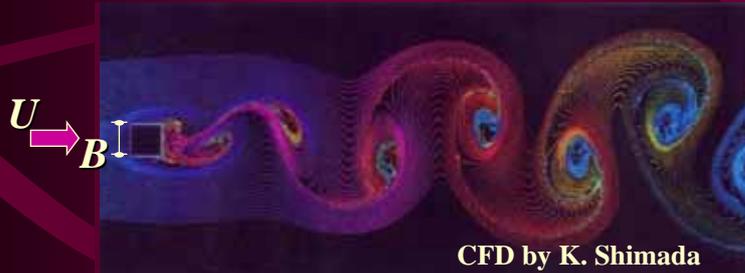
$$F_L = \rho U \Gamma$$

Kutta-Joukowski's Theorem

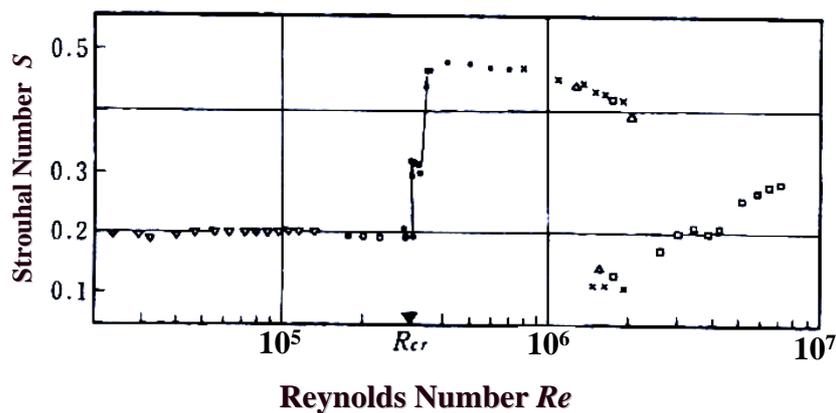
Periodic lateral force due to alternate vortex shedding

$$f_v = S \frac{U}{B}, \quad S = \frac{f_v B}{U}$$

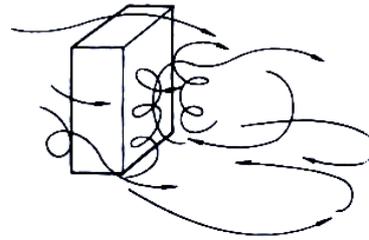
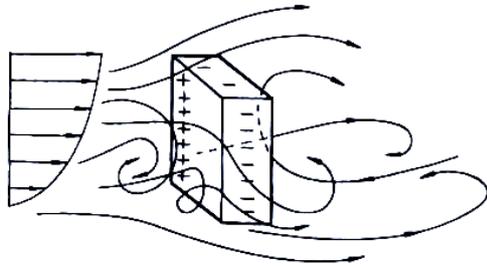
- f_v : Shedding frequency of Karman vortices
- S : Strouhal number
- U : Wind Speed
- B : Reference Length



Vortex shedding and Strouhal Number



Variation of Strouhal number with Reynolds number
(Circular cylinder, 2D, Uniform flow, Shewe 1983)

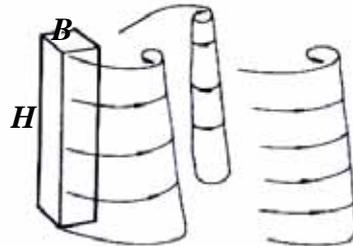
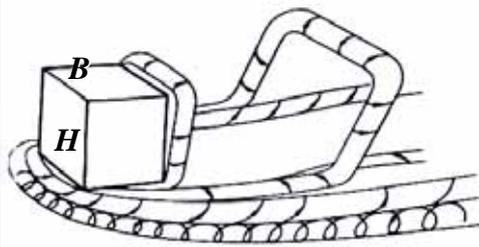


(a) front side

(b) rear side

Flow around a building

(T. Tamura)



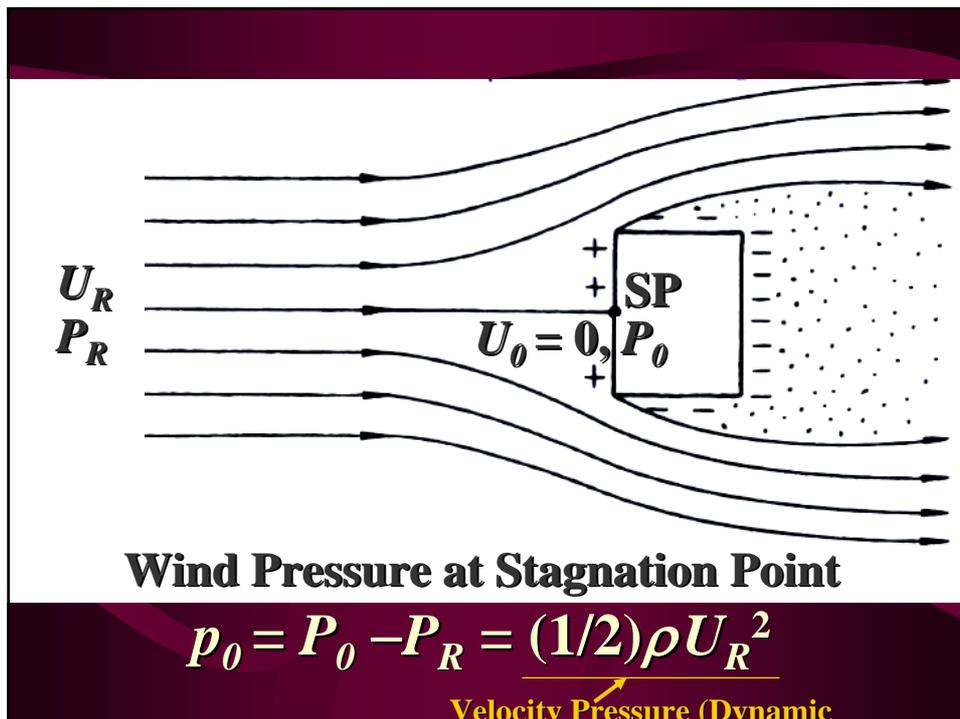
(a) Small H/B (Aspect Ratio)

(b) Large H/B

Vortex formation behind building models

Static Wind Forces

Velocity Pressure and Wind Pressure Coefficient

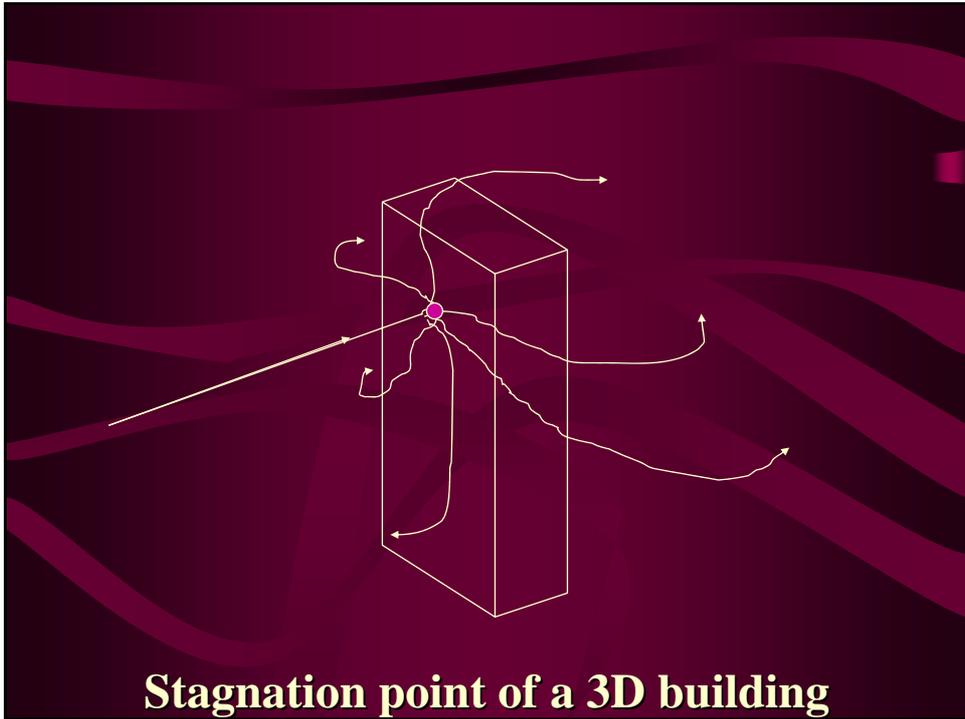


■ **Wind Pressure Coefficient**

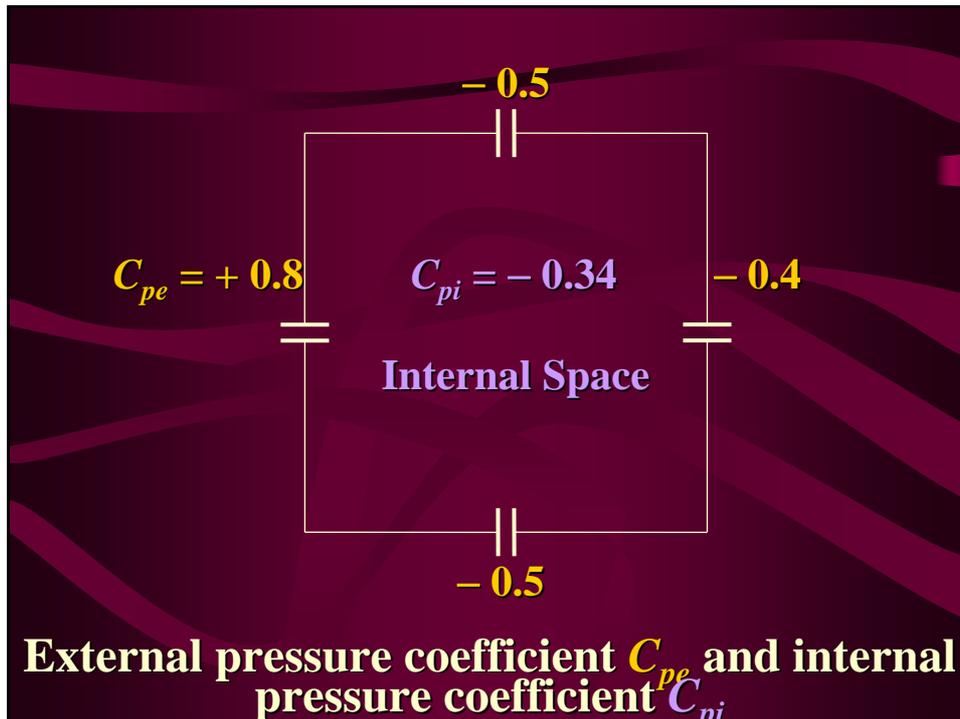
$$C_{pk} = \frac{p_k}{\frac{1}{2} \rho U_R^2} \leftarrow \text{Velocity Pressure : } q_R$$

$$p_k = P_k - P_R$$

p_k : Wind pressure at point k
 P_k : Static pressure at point k
 P_R : Reference static pressure
 ρ : Air density
 U_R : Reference wind speed



Internal Pressure Coefficient and External Pressure Coefficient

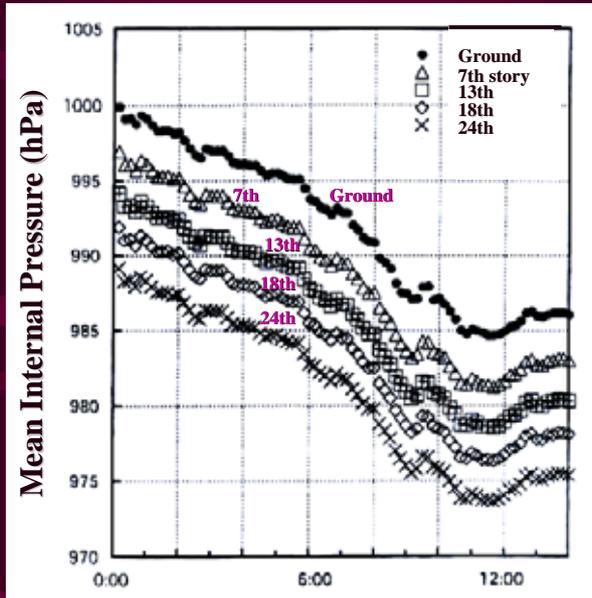


Equilibrium Equation of Flows Through Gaps:

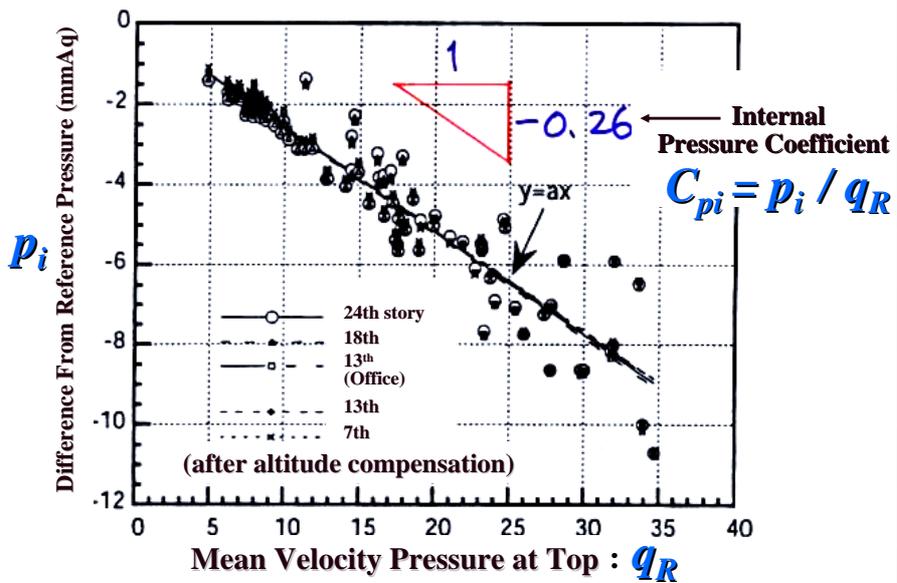
- The flow is proportional to the velocity at each gap.
- The velocity is assumed to be proportional to the square root of the pressure difference between the gap.

$$\sqrt{|0.8 - C_{pi}|} = 2\sqrt{|C_{pi} + 0.5|} + \sqrt{|C_{pi} + 0.4|}$$

$$\rightarrow C_{pi} = -0.34$$

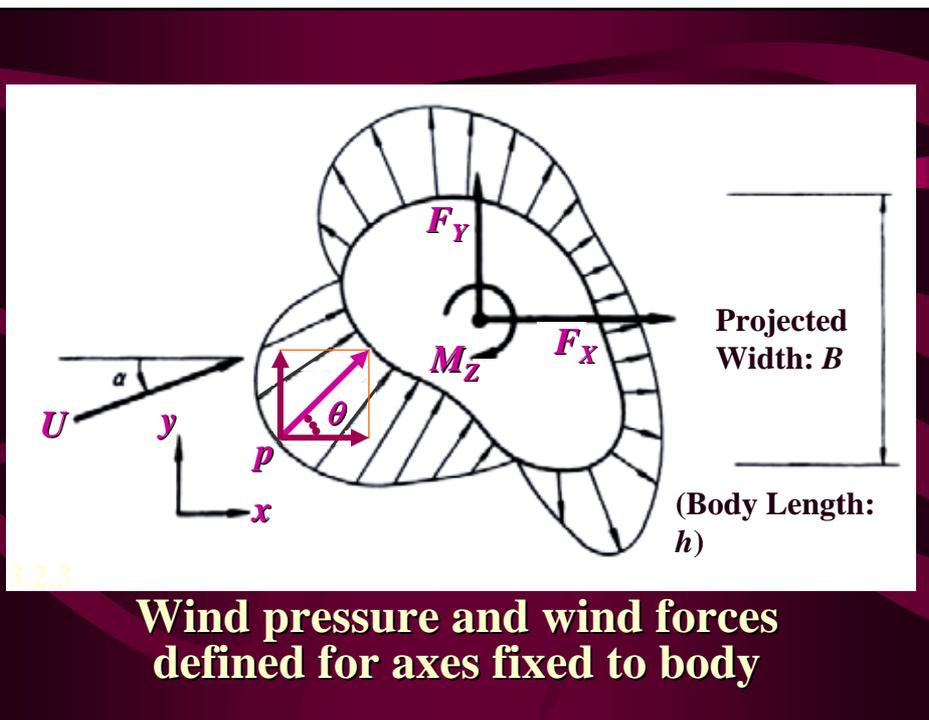


Temporal variations of mean internal pressures (Full-scale, Kato et al., 1996)



Variation of mean internal pressures with reference velocity pressure (Full-scale, Kato et al., 1996)

Wind Force Coefficient



■ Wind Forces

$$F_X = \int_S p \cos \theta h ds$$

$$F_Y = \int_S p \sin \theta h ds$$

h : Body length

■ Wind Force Coefficients

$$C_{FX} = \frac{F_X}{\frac{1}{2} \rho U_R^2 A}, \quad C_{FY} = \frac{F_Y}{\frac{1}{2} \rho U_R^2 A}$$

A : Projected area = Bh

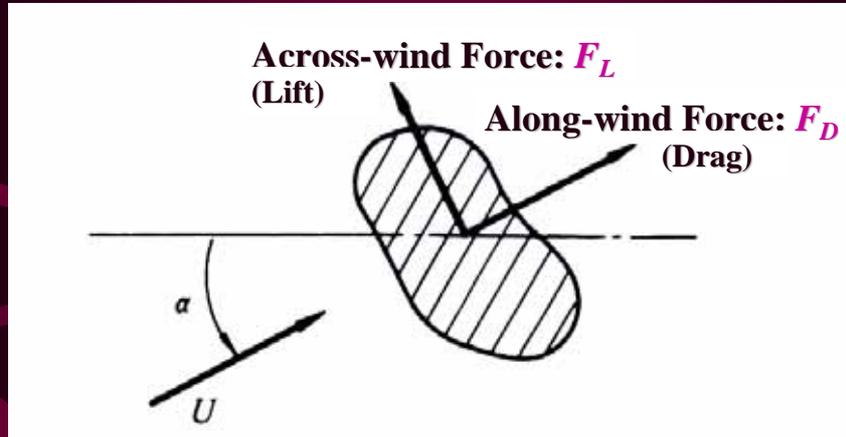
Aerodynamic Moment Coefficients

$$C_{MZ} = \frac{M_Z}{\frac{1}{2} \rho U_R^2 AL}$$

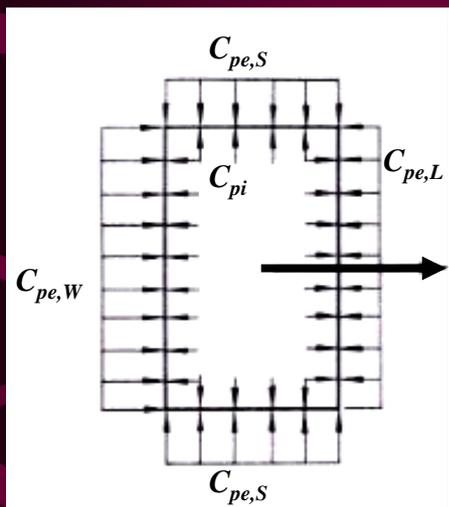
M_Z : Aerodynamic Moment

A : Projected area

L : Reference Length



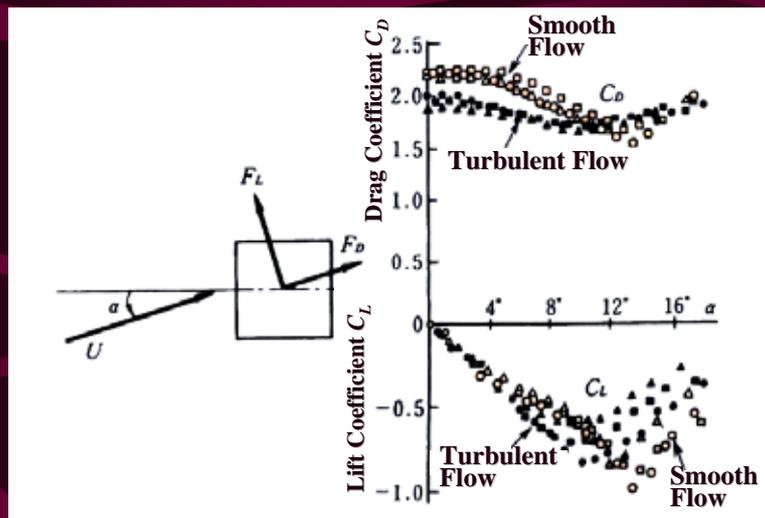
Along-wind force and across-wind force defined for the axes fixed to wind



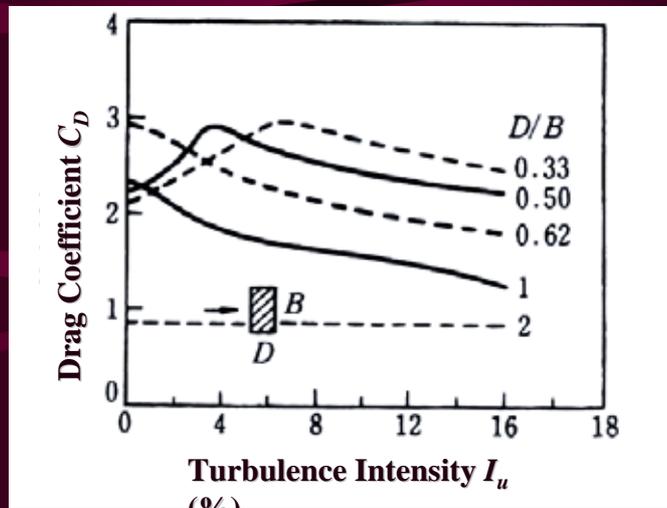
Along-wind Force Coefficient
 $C_F = C_{pe,W} - C_{pe,L}$

Wind pressure coefficient and wind force coefficient for a building with an internal space

Static Wind Forces Acting On Bluff Bodies

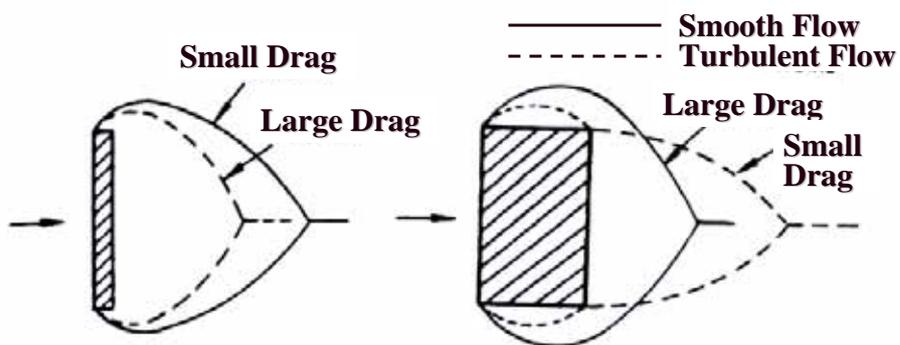


Variations of mean drag and lift coefficients with attacking angle (2D)

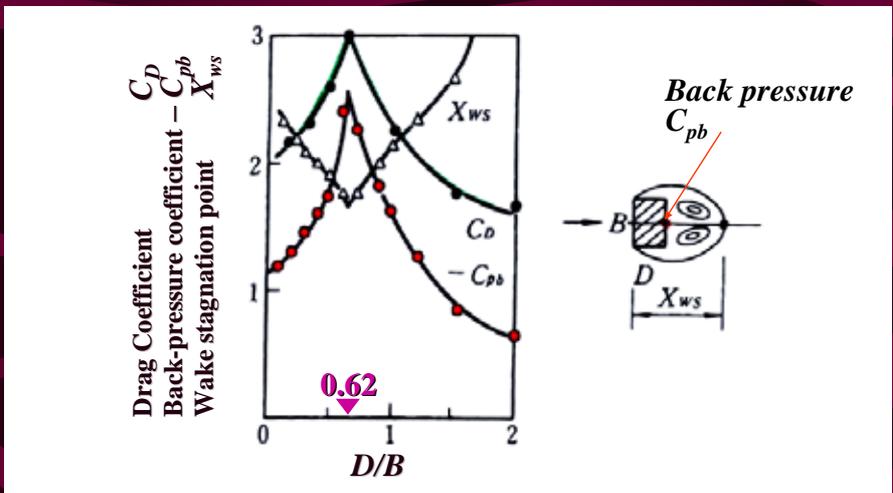


Effects of turbulence on mean drag coefficients of 2D prisms

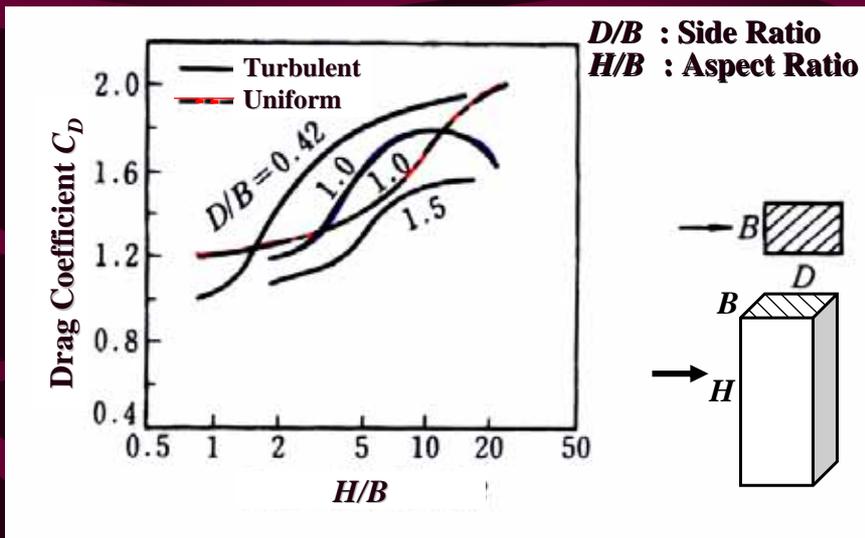
Entrainment effects of turbulence



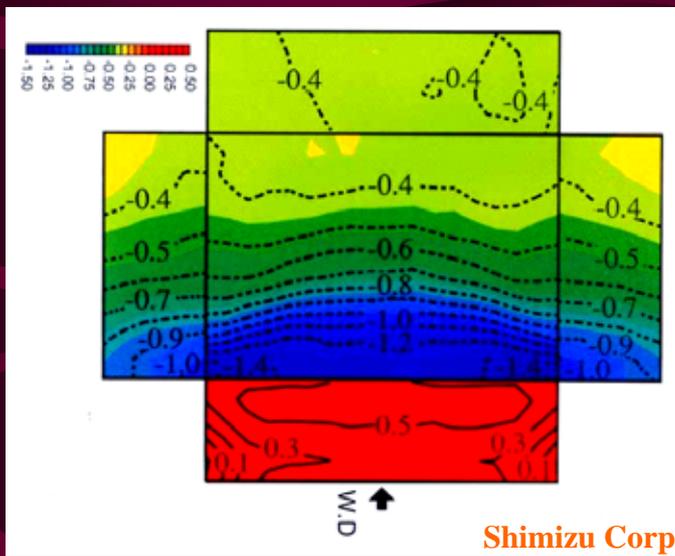
(a) Flat plate ($D/B = 0.1$) (b) Prism ($D/B = 0.5$)
Effects of turbulence on separated shear layers (Laneville et al., 1975)



Variation of mean drag coefficient, back-pressure coefficient, and wake stagnation point with side ratio (2D, Uniform flow)

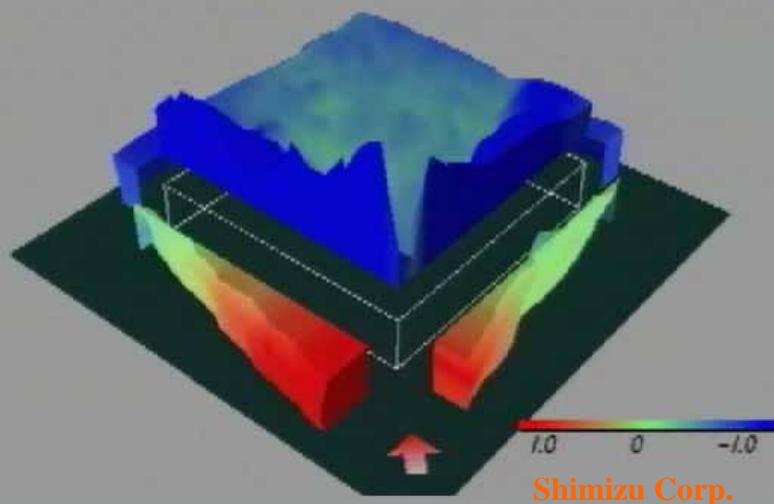


Variation of mean drag coefficients of 3D rectangular prisms with aspect ratio

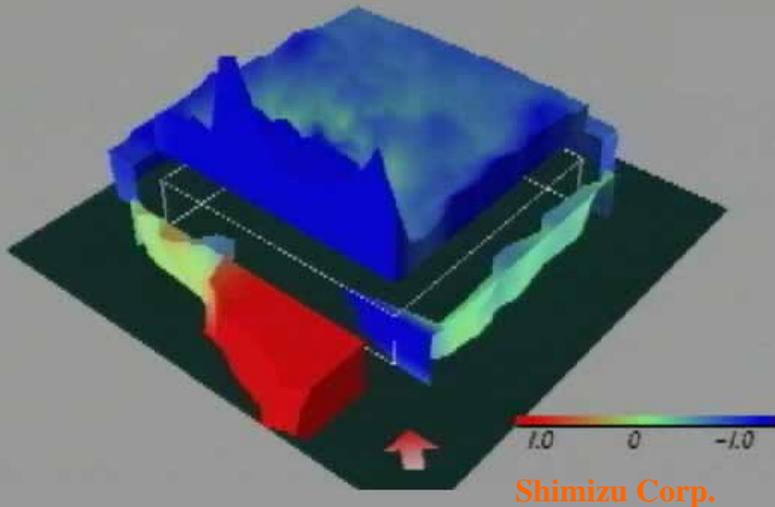


**Mean pressure distribution
on a low-rise building model**

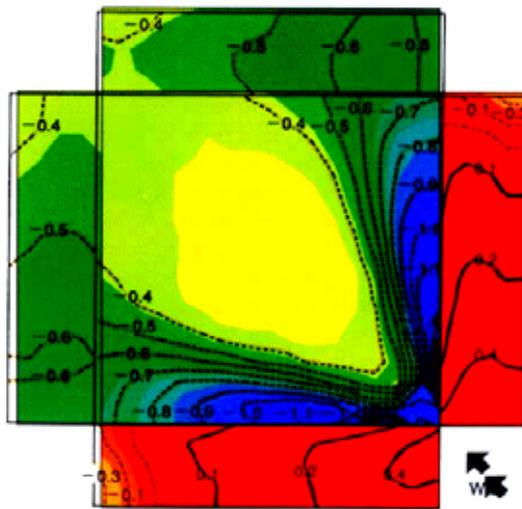
**Instantaneous pressure distribution
on a low-rise building model (45° Wind)**



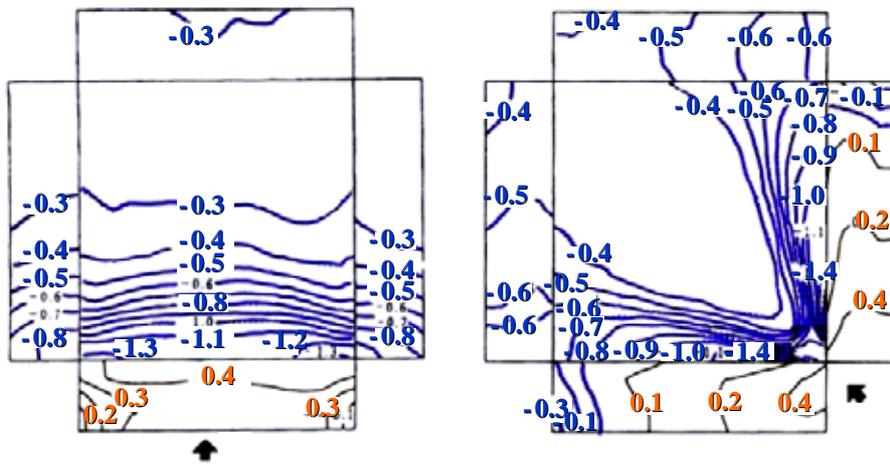
**Instantaneous pressure distribution
on a low-rise building model (45° Wind)**



Conical Vortices



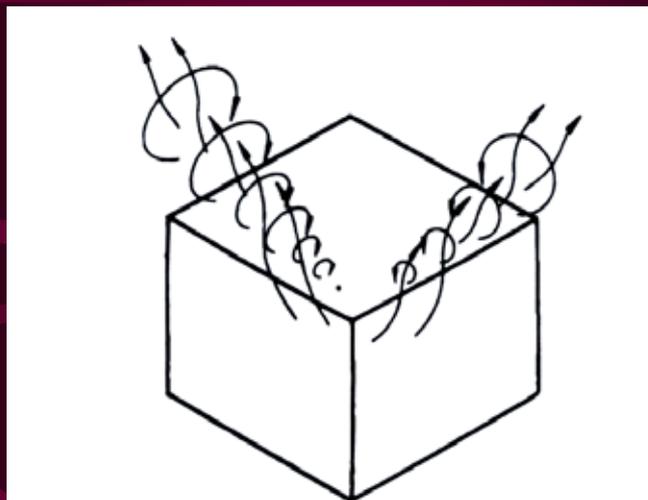
**Mean pressure distributions Shimizu Corp
on a low-rise building model (45° Wind)**



(a) 0° Wind

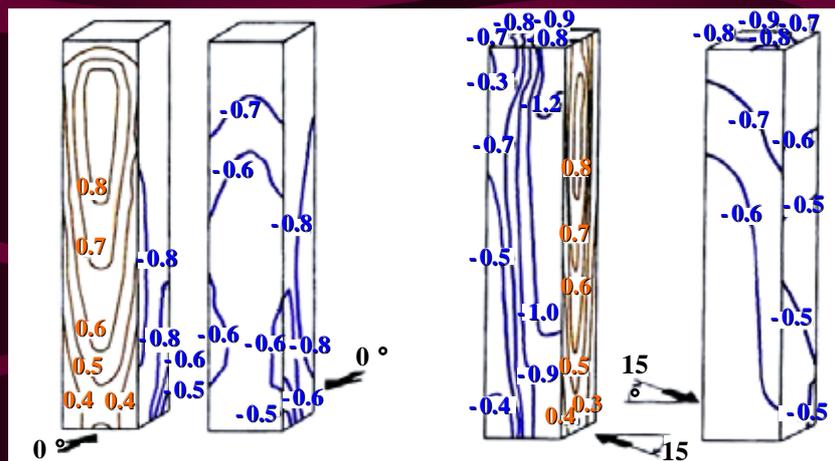
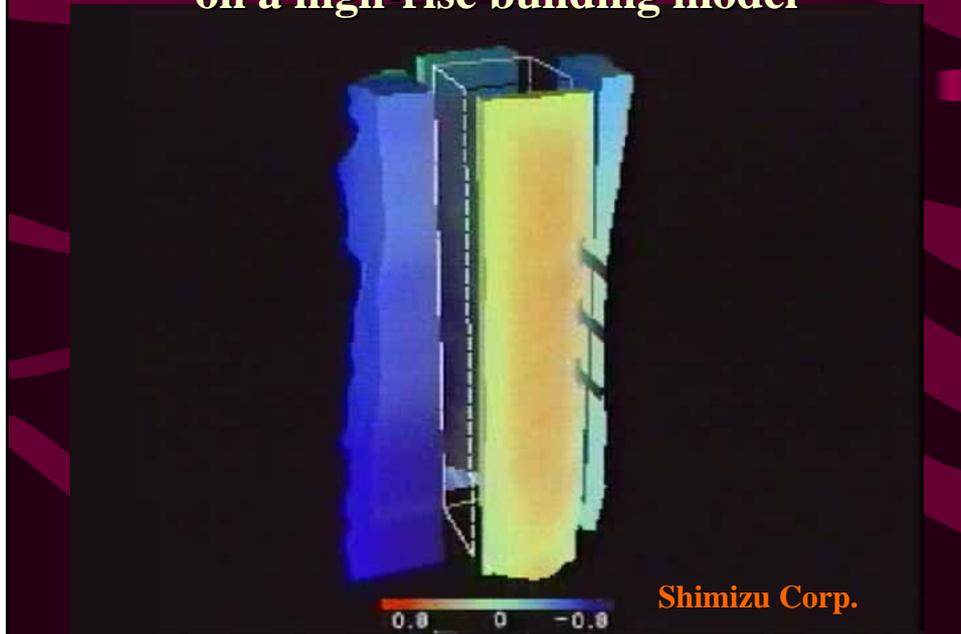
(b) 45° Wind

**Mean pressure distributions
on a low-rise building model**



**Conical vortices generated
from corner eaves**

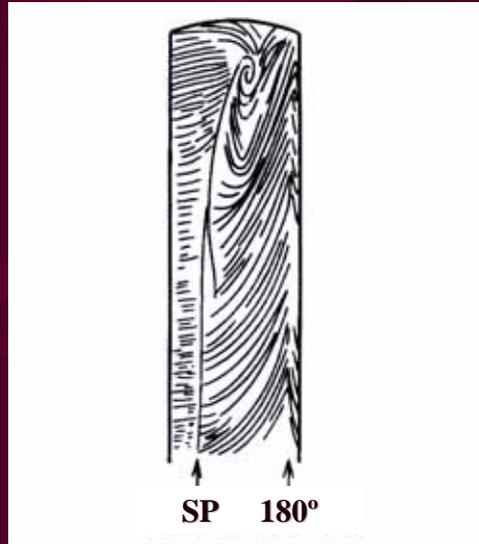
Instantaneous pressure distribution on a high-rise building model



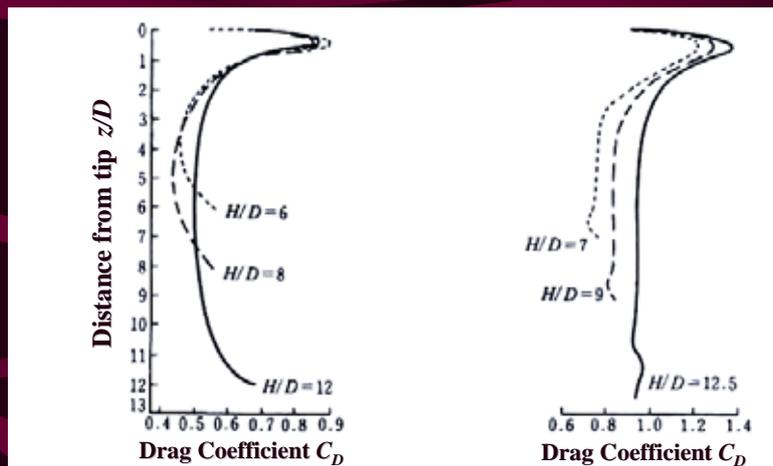
(a) 0° Wind

(b) Glancing Wind

Mean pressure distributions on a high-rise building model

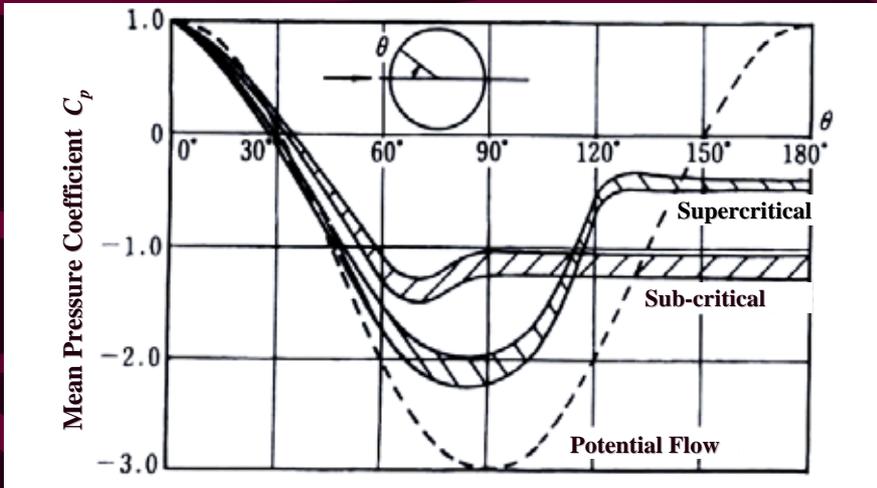


Surface flow pattern near the tip of 3D circular cylinder (Oil film technique, view obliquely from behind, Lawson, 1980)

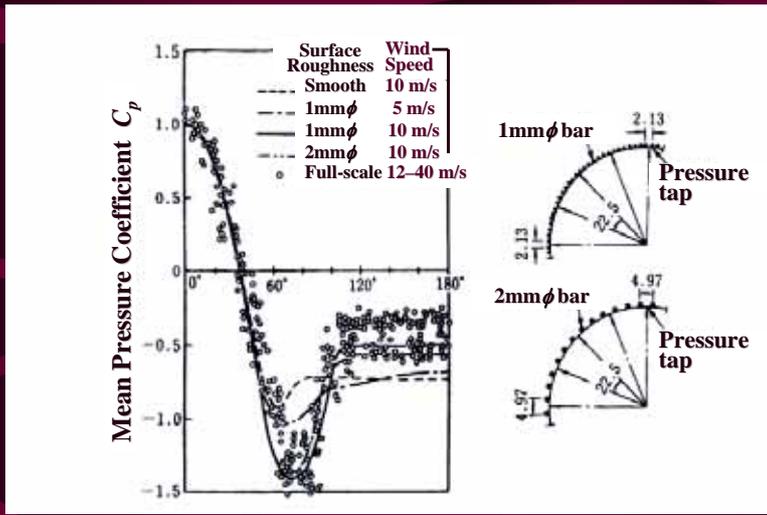


(a) Post-supercritical Regime $Re = 2.7 \sim 5.4 \times 10^6$ (b) Sub-critical Regime $Re = 1.33 \times 10^4$

Vertical distributions of local mean drag coefficients of 3D circular cylinders



Mean pressure distribution on a circular cylinder (Sach, 1972)



Mean pressure distributions on a full-scale chimney and wind tunnel models with different surface roughnesses